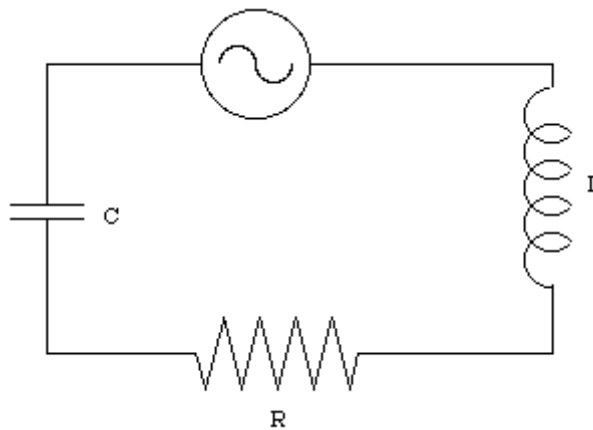


## ***Resonance in AC Circuits***



***Produced by the Physics Staff at  
Collin County Community College***

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## Purpose

In this experiment, you will investigate electromagnetic induction in solenoids and paired wire coils. You will also investigate electromagnetic permeability by noting the effects of different materials on the induced magnetic field of a coil.

## Equipment

- Power Amplifier
- Voltage Sensor
- Two long Patch cords
- Digital Multimeter
- *LRC* Circuit Board
- Graph paper
- Three short Patch Cords

## Theory

The most significant feature of a series *LRC* circuit is resonance. For a fixed value of applied AC voltage, the current in the circuit and the voltage across all three components vary with the frequency of the applied voltage. There is a particular frequency at which the current is a maximum and the voltages across *L* and *C* are equal. This phenomenon is called *resonance*, and the frequency at which it occurs is called the resonant frequency of the circuit.

As in any circuit, the current *I* through a series *LRC* circuit depends on the applied voltage  $V_0$  and the circuit impedance *Z*. The relationship is given by

$$I = \frac{V_0}{Z} \quad \text{Equation 10.1}$$

The impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{Equation 10.2}$$

where  $X_L$  and  $X_C$  are the reactance for the inductor and the capacitor, respectively. The inductive reactance is given by

$$X_L = \omega L \quad \text{Equation 10.3}$$

and the capacitive reactance is given by

$$X_C = \frac{1}{\omega C} \quad \text{Equation 10.4}$$

where  $\omega = 2\pi f$ , and *f* is the frequency of the applied voltage. Since the reactance depends on the frequency, the impedance and the current also depend on the frequency.

From Equation 10.1, the current will be a maximum when the impedance is a minimum. From Equation 10.2, the impedance will be a minimum when the two reactance values are equal and cancel each other. The reactance values will be equal when:

$$\omega L = \frac{1}{\omega C}$$

and  $\omega_0 = \frac{1}{\sqrt{LC}}$  *Equation 10.5*

This value of  $\omega_0$  is the resonant angular frequency. At this frequency, the impedance is equal to the resistance,  $Z = R$ , but at all other frequencies,  $Z > R$ . The impedance has a minimum value at resonance, and the current has a maximum value.

## **Procedure**

In this experiment the Power Amplifier applies an AC sine wave voltage to a series *LRC* circuit. You will use the Pasco voltage sensors to measure the voltages across the various components of the circuit. The Signal Generator will control the frequency and amplitude of the applied voltage.

In Part A you will measure the voltage across the resistor over a range of frequencies above and below the resonant frequency of the circuit. You will then plot the normalized voltage across the resistor versus the frequency. The normalized voltage is the ratio of the voltage you measure across the resistor to the voltage you measure across the output of the Power Amplifier.

The voltage versus frequency plot shows the resonance phenomenon. By taking closely-spaced voltage measurements near resonance, you can determine the resonant frequency quite accurately from the graph.

In Part B you will measure circuit voltages at three widely different frequencies: one that is much lower than the resonant frequency, one at the resonant frequency, and one that is much higher. At each of these three frequencies, you will measure the voltage across the Power Amplifier, the inductor, the capacitor, and the resistor. You will calculate the vector sum of the voltages across *R*, *L*, and *C* at each frequency, and compare this sum to the voltage output by the Power Amplifier.

In Part C you will display the instantaneous voltage across the resistor versus the instantaneous voltage output by the Power Amplifier on the Scope display. This plot of one sine wave vs. another sine wave creates a Lissajous figure. In general it is an ellipse, but at resonance the ellipse degenerates into a straight line. This provides a very accurate method of determining the resonant frequency.

### **A. Determining Resonant Frequency from $V_R$ vs. $f$ Plot**

#### ***Equipment Setup***

The Pasco *LRC* circuit board contains various inductors, resistors, capacitors and other components. Each component on the circuit board except the inductor is labeled with its type and value. There are banana jacks for wiring the components to each other and to external equipment.

1. Use the patch cords to connect the inductor (coil of wire), the 10  $\Omega$  resistor and the 100  $\mu\text{F}$  capacitor in series with the output of the Power Amplifier. ***Do not insert the iron core into the coil.*** Using the multimeter, measure the combined resistance of the resistor and inductor. This is your total resistance *R*. Record the values of the circuit resistance *R*, capacitance *C*, and inductance *L* in Table 10.1.

2. Connect a voltage sensor across the resistor to Channel B of the Interface, and connect another voltage sensor across the power amplifier to Channel C.
3. Open Data Studio, select Create experiment, select the Power Amplifier, and select the Voltage sensor twice.
4. Double-click on each Voltage sensor icon to open its Sensor Properties window. Set the Sample Rate to 1,000 Hz for each sensor.
5. In the Signal Generator window, select the sine wave; and set the Amplitude to 3.000 volts. Record this voltage in Table 10.1.
6. Open a Scope display and select Voltage, Ch B (V). Then open another Scope display and select Voltage, Ch C (V).

### ***Data Collection and Analysis***

7. Switch on the Power Amplifier.
8. Set the Signal Generator frequency to 40.000 Hz.
9. Record the resistor voltage  $V_R$  (peak voltage from the scope display) and the frequency  $f$ . Calculate and record the normalized voltage  $V_N$  in Table 10.1.
10. Increase the frequency by 20 Hz.
11. If the frequency is not greater than 300 Hz, repeat steps 10 and 11.
12. Without disassembling your experimental setup, create a graph (in Excel) of normalized voltage  $V_N$  vs. frequency  $f$ . Locate the approximate frequency at which the normalized voltage is a maximum. This is an initial estimate of the resonant frequency of the circuit.
13. Select a frequency that is about 20 Hz lower than your estimated resonant frequency.
14. Repeat step 10, and increase the frequency by 2 Hz.
15. Repeat step 15 until the frequency is about 20 Hz more than your estimated resonant frequency. Adjust the number of rows in Table 10.1 as needed.
16. Add these new data points to the normalized voltage vs. frequency graph. You should now be able to find the resonant frequency from your graph more accurately. Connect the plotted data points with a smooth curve to obtain a single graph of the normalized voltage versus frequency. Find the resonant frequency from your graph, and record it in Table 10.1.

### **B. Total Voltage Measurements**

These measurements use the same equipment setup as for Part A.

### ***Data Collection and Analysis***

1. Set the Power Amplifier to a frequency well below the resonant frequency.
2. Using the voltage sensors (and the scope displays), measure and record (in Table 10.2) the voltage across  $R$ , the voltage across the power amplifier, and the frequency.
3. Connect the voltage sensors across the capacitor and the inductor. Measure and record the voltages across the capacitor and the inductor.

4. Set the Power Amplifier to the resonant frequency. Repeat steps 2 and 3.
5. Set the Power Amplifier to a frequency well above the resonant frequency. Repeat steps 2 and 3.
6. For each frequency calculate the magnitude of the *vector sum* of the voltages across the inductor, the capacitor and the resistor. Compare this sum to the voltage across the power amplifier by calculating a percent difference.

### **C. Measuring Phase Difference**

#### ***Equipment Setup***

Switch off the power amplifier. Connect the voltage sensors across the resistor and the power amplifier. Open a new Scope display and select Channel B as the vertical axis input. Drag Channel C from the Data listing to the horizontal axis of the Scope display. The display will be a plot of the instantaneous voltage across the resistor versus the instantaneous voltage applied to the circuit (by the power amplifier).

This type of plot is called a Lissajous figure. For this case, where the frequencies of the two voltages are the same, the general shape of the figure is an ellipse. When the two voltages are in phase (have the same phase), the shape is a straight line at  $45^\circ$ . By varying the frequency you can alter the phase relationship between the two voltages and this is reflected in the shape of the displayed figure.

#### ***Data Collection and Analysis***

1. Switch on the power amplifier.
2. Select a frequency well below the resonant frequency, and observe the shape of the Lissajous figure.
3. Increase the frequency slowly and observe the changes in the figure.
4. Carefully adjust the frequency near the resonant frequency to obtain a single straight line (note that the figure is a straight line at the resonant frequency). Once the resonant frequency has been found, record it in Table 10.3.
5. Calculate and record the percent difference between the resonant frequencies found in Parts A and C.