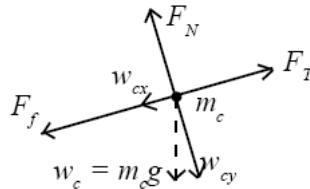
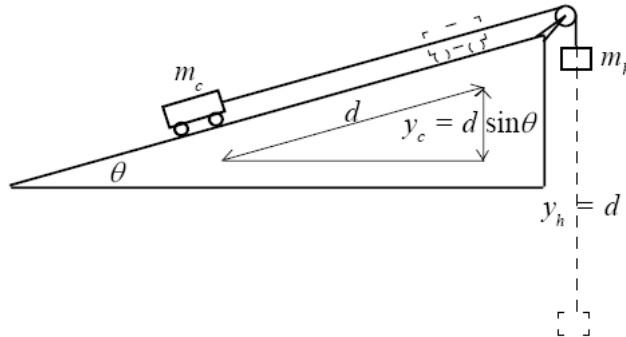


## Work & Mechanical Energy



*Produced by the Physics Staff at  
Collin County Community College*

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## **Purpose**

In this experiment, you will investigate the relationship between work and energy

## **Equipment**

- 1 Track w/ Leveling Feet
- 1 End Stop
- 1 Mass Set w/ Hanger
- 1 Plunger Cart and Mass
- 1 Lab Balance
- 1 Pivot Clamp and Ringstand
- 1 Smart Pulley w/ Clamp
- 1 Meter stick
- 1 Roll of String

## **Introduction**

This experiment is about an abstract idea — a mathematical concept — which is this: There exists a numerical quantity, a number we can calculate, whose value remains the same whenever anything in nature happens. The quantity does not describe a mechanism, or anything concrete. It's just a strange fact of nature that we can calculate this number, and after we observe nature go through her tricks and then calculate the number again, it has the same value.

It's something like the white bishop in chess. After any number of moves – details unknown – it is always on a white square. It is a law of chess.

We call this numerical quantity *energy*, and it is an omnipresent property of objects and systems. But as important as it is, we cannot measure it; we can only calculate it.

There is another concept, work, that is clearly related to energy in that an object or system that possesses energy has the capability to do work. An object or system can *possess* energy, but it *performs* work. Work is, therefore, *energy in transit* into or out of an object or system.

A system gains energy when its surroundings perform work on it (such work is considered to be positive since it results in an increase of energy), and it loses energy when it performs work on its surroundings (this work is considered to be negative).

Like its energy, the amount of work a system performs can be calculated but not measured. It is the scalar product of the force the system applies on an object times the resulting displacement of that object. Any applied force that causes an object to move (to be displaced) performs work on it. Force and displacement, then, are the two essential elements of work.

The energy possessed by a system appears in various forms. All physical or chemical activities in nature involve the conversion of energy from one form to another. Inventors have developed many types of systems whose purpose is to convert energy from one form to another. In a mechanical system (called a *machine*), energy is converted between its kinetic, potential, and thermal forms.

The strange law of nature mentioned in the first paragraph is this: when we consider energy in all its forms, we find that the total energy associated with any isolated system is conserved. That is, energy is never created or destroyed in any system, it is only changed into another form.

In an ideal (or conservative) system, e.g., a machine running without friction, energy is converted back and forth between its kinetic form (energy due to its speed) and its potential form (due to its position). Physicists have observed that the total *mechanical energy* in an isolated system (the sum of its instantaneous kinetic and potential energies) stays constant. This observation is expressed as the Law of Conservation of Mechanical Energy.

In real systems, however, friction is always present, and so real machines are non-conservative. They lose some mechanical energy as a result of the work the machine does against frictional forces. Even so, the total energy of the system is still conserved, the mechanical energy consumed by friction is simply converted into thermal energy – a form of energy we call heat.

You can calculate the values of work and energy from measurements of such properties as mass, position, displacement, speed, and force. Using known relationships between work and various forms of energy, you can calculate important system quantities that are otherwise too difficult to measure.

In this experiment, you will investigate the relationship between work and energy and thereby learn to:

1. Explain how work and energy are related.
2. Describe how the work done by frictional forces can be determined experimentally from measurements of force and displacement.
3. Describe the differences between conservative (ideal) and non-conservative (real) systems, and the difference between the conservation of *mechanical* energy in an ideal system and the conservation of *total* energy in a real system.

## **Theory**

If you apply a force to an object of a given mass and if the object moves a given distance, your force performs work. Work done *on* an object increases the object's energy; work done *by* an object on its environment decreases its energy. Your work on the object is transformed into three possible forms of energy.

If your work is against *only* the object's inertia (no friction, no height increase), its speed will increase, and the amount of work performed will be equal to the object's increase in kinetic energy  $\Delta K$ :  $W = \Delta K$

The object's kinetic energy is related to its mass  $m$  and speed  $v$  by

$$K = \frac{1}{2}mv^2$$

$$\text{So } W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$$

If **your** work is against *only* gravity (no acceleration, no friction), the object's vertical position (height) will increase, and the amount of work performed will be equal to the object's increase in potential energy  $\Delta U$ :

$$W = \Delta U$$

The object's potential energy is related to its mass and height  $h$  by

$$U = mgh$$

$$\text{So } W = \Delta U = mgh_f - mgh_i = mg(h_f - h_i)$$

If your work is against *only* friction (no acceleration, no height increase), the object's temperature will increase, and the amount of work performed will be equal to the object's increase in heat energy  $\Delta H$ :  $W = \Delta H$

The object's heat energy is related to its mass, its composition, and its change in temperature, a relationship you will explore further later this semester.

In all these cases, the work done on the object by an external force is given by

$$W = \vec{F} \cdot \vec{d} = Fd \cos \varphi$$

where  $d$  is the magnitude of the displacement, and  $\varphi$  is the angle between the vectors  $\vec{F}$  and  $\vec{d}$ .

Using this work-energy relationship, we can combine the four equations above to find the amount of heat generated by friction in terms of the work done on the object by an applied force  $F_a$ , its change in speed, and its change in height:

$$\Delta H = F_a d \cos \varphi - \frac{1}{2} m(v_f^2 - v_i^2) - mg(h_f - h_i)$$

For such a system, you can calculate the heat generated ( $\Delta H$ ) in three ways:

1. When the object's speed and height remain fixed (the system's total mechanical energy is constant), you can eliminate  $W$  to get

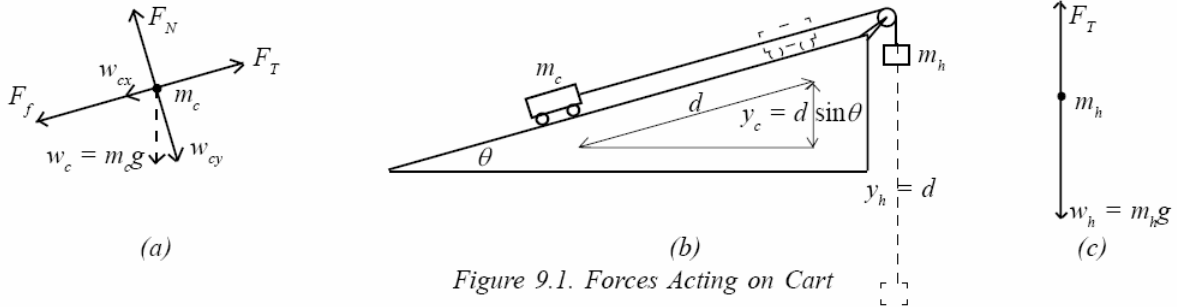
$$\Delta H = F_a d \cos \varphi$$

2. When only the object's height remains fixed (its potential energy is constant), you can eliminate  $W$  to get

$$\Delta H = F_a d \cos \varphi - \frac{1}{2} m(v_f^2 - v_i^2)$$

3. When only the object's speed remains fixed (its kinetic energy is constant), you can eliminate  $W$  to get

$$\Delta H = F_a d \cos \varphi - mg(h_f - h_i)$$



Consider a cart of mass  $m_c$  being pulled up a track inclined at an angle  $\theta$ , as illustrated in Figure 9.1(b). All the forces on the cart  $m_c$  and on the hanging mass  $m_h$  are illustrated in free-body diagrams in Figures 9.1(a) and 9.1(c).

To apply Newton's second law to the cart, resolve the forces into components parallel to the track (the  $x$  axis) and normal to it (the  $y$  axis). There are three forces exerted on the cart along this  $x$  axis: one directed up the track,  $F_T$ , and two others directed down the track,  $w_{cx}$  and  $F_f$ .  $F_T$  is applied by the tension in the string,  $w_{cx}$  is the component of the cart's weight parallel to the track,  $w_{cx} = m_c g \sin \theta$ , and  $F_f$  is the frictional force on the cart. This frictional force will be static ( $F_{sf}$ ) if the cart is at rest, and will be kinetic ( $F_{kf}$ ) if the cart is moving up the track. However, if the cart is moving down the track,  $F_{kf}$  will be directed up the track because kinetic frictional force is always opposite to the velocity. The magnitude of either frictional force is given by

$$F_f = \mu F_N$$

There are two forces exerted on the cart along the  $y$  axis: the normal force by the track,  $F_N$  and the  $y$  component of the cart's weight,  $w_{cy} = m_c g \cos \theta$ .

Applying the second law to the cart, we get

$$\sum F_x = F_T - m_c g \sin \theta - \mu_{kf} F_N = m_c a$$

$$\sum F_y = F_N - m_c g \cos \theta = 0$$

The second equation tells us that  $F_N = m_c g \cos \theta$ , so the frictional force can be re-written:

$$F_{kf} = \mu_{kf} m_c g \cos \theta$$

There are only two forces exerted on the hanging mass, both in the vertical direction. The tension force  $F_T$  pulls up on it and its weight  $w_h$  pulls down on it. So for this mass, we get

$$\sum F_x = 0$$

$$\sum F_y = F_T - w_h = F_T - m_h g = -m_h a$$

The work done by the hanging mass as it falls is equal to the product of  $(F_T - m_h g)$  and  $d$ . To perform this amount of work, the hanging mass must lose potential energy – its height must decrease. The work it performs is split three possible ways. Some of it can increase the cart's kinetic energy (its speed), some can increase its potential energy (its height), and some, because of friction, is always transformed into heat.

In all three cases defined by these equations, the amount of work converted to heat can also be thought of as work performed on the cart by only the frictional force:

$$\Delta H = \vec{F}_{kf} \cdot \vec{d} = -F_{kf} d$$

Such frictional work is always negative – the cart always loses some mechanical energy – because  $\vec{F}_{kf}$  and  $\vec{d}$  are always in opposite directions.

Let us examine each of the cases in more detail. If the track is horizontal so that the cart's potential energy remains fixed, and if hanging mass is just sufficient to pull the cart at constant speed so that its kinetic energy also remains fixed, the frictional heat generated is given by  $\Delta H = F_a d \cos \varphi$ . Since  $F_a = F_T = m_h g$ ,  $d = y_h$ , and  $\varphi = 0$ , the heat generated can be written as:

$$\Delta H = m_h g y_h$$

and all the terms on the right side can be measured.

In the second case, the track is still horizontal but the hanging mass is greater so that the cart accelerates, the frictional heat generated is given by  $\Delta H = F_a d \cos \varphi - \frac{1}{2} m (v_f^2 - v_i^2)$ . If the cart starts from rest,  $v_i = 0$ , and the equation can be re-written as

$$\Delta H = m_h g y_h - \frac{1}{2} m_c v_f^2$$

and again, all the terms on the right side can be measured.

In the third case,  $\theta \neq 0$  so the track is inclined, and the cart's potential energy increases. If the hanging mass is adjusted so that the cart moves up at constant speed, its kinetic energy remains fixed, and the frictional heat generated is given by  $\Delta H = F_a d \cos \varphi - m g (h_f - h_i)$ .

This equation can be re-written as

$$\Delta H = m_h g y_h - m_c g y_c = m_h g y_h - m_c g y_h \sin \theta$$

and once again, all the terms on the right side can be measured.

You might assume that if the cart moves *down* the track at the same constant speed, the magnitude of the frictional force would be the same as it was going up. You will investigate this assumption experimentally.

## **Procedure**

You will use the Smart Pulley to measure the cart's speed. Using the methods described in the theory section, you will calculate the amount of heat generated by friction when the cart is pulled on the track by the hanging mass. Plug the Smart Pulley sensor into Digital Channel 1 on the Pasco Interface.

### **A. When Both $U$ and $K$ are Constant**

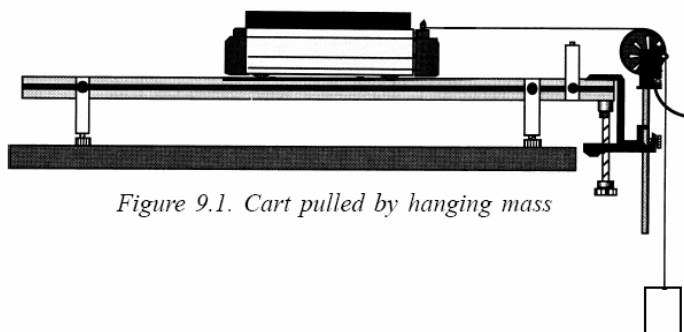


Figure 9.1. Cart pulled by hanging mass

1. Fasten the two sets of leveling feet about 50 cm in from each end of the track, and position the track with its right end projecting over the end of the table. Place the cart on the track and adjust the leveling feet so that the cart doesn't roll on its own one way or the other, as shown in Figure 9.1.
2. Mount the bumper a few inches in from the right end of the track, and attach the Smart Pulley to the end using the Universal Table Clamp.
3. Measure and record the mass of the empty cart  $m_{ec}$ .
4. Cut a length of string (about  $1\frac{1}{2}$  m should do) and tie a small loop at each end. Hook one end over the plunger lock of the cart and hook a mass hanger at the other end. Thread the string over the pulley, ensuring that it doesn't rub anywhere.
5. Switch on the computer system, open Data Studio, and select Create Experiment. Assign the *Smart Pulley* sensor to the Interface. Double-click on the sensor icon to open the Sensor Properties window. Select the *Measurement* tab, select *Velocity*, Ch. 1 (m/s), and click OK.
6. Open a *Graph* display. This display should show velocity vs. time.
7. Add a mass  $m_w$  of 1000 g to the top of the cart. Record the total mass of the cart  $m_c = m_{ec} + m_w$  in Table 9.1.
8. Pull the cart to the left until the mass hanger is just below the clamp. Mark the position of the front wheels and adjust the pulley height so that the string between the cart and the pulley is parallel to the track. (Measure it to be sure.)

9. Place just enough mass on the hanger to pull the cart at **constant speed** after you tap it to overcome static friction. Before you release the cart, press *Start* in Data Studio. When the cart strikes the bumper, press *Stop*. Use the graph in Data Studio to determine when the cart's speed is constant (when the speed is constant, the graph will be a horizontal line). If the empty hanger is too heavy, replace it with a paper clip bent to be a mass hanger. If the hanger doesn't hit the floor before the cart hits the bumper, replace the string with a longer piece and remark the starting position of the cart.
10. Once you have achieved a constant speed for the cart, pull the cart to its start position and hold it in place. Use a meter stick to measure the height of the bottom of the mass hanger  $y_h$ . Record the total hanging mass  $m_h$  and the initial height of the hanger  $y_h$  in Table 9.1.
11. Calculate the heat  $\Delta H$  generated by friction and record it in Table 9.1.
12. Repeat steps 7-11, increasing  $m_w$  by 200 g each trial (such that on the 5<sup>th</sup> trial,  $m_w = 1800$  g), for a total of five trials. Record your data in Table 9.1.

### **B. When Only $U$ is Constant**

13. Place a total mass  $m_w$  of 1000 g on top of the cart. Record the total mass of the cart  $m_c = m_{ec} + m_w$  in Table 9.2. Place a total mass  $m_h$  on the hanger such that, when released, the cart moves down the track at a constant speed (just use the same value of  $m_h$  from part A).
14. Now add 20 g to the hanger to make the cart accelerate.
15. Place the cart at its starting position from part A. Press *Start* in Data Studio, and release the cart. When the cart strikes the bumper, press *Stop*.
16. Click the *Smart Tool* button and measure the velocity at the top of the linear portion of the graph (this is the final velocity of the cart  $v_f$ ). Record this value in Table 9.2.
17. Calculate the heat  $\Delta H$  generated by friction (the distance  $y_h$  should be the same as in Part A) and record it in Table 9.2.
18. Repeat steps 15-17 four times, adding an additional 20 g to the hanger for each trial, for a total of five trials. Record your data in Table 9.2.

### **C. When only $K$ is Constant**

19. Install the pivot clamp on the back side of the track about 50 cm from the end with the Smart Pulley. Use the ringstand to raise the pulley end of the track such that the inclination angle  $\theta$  is  $10^\circ$ . Record this angle in Table 9.3.
20. Place a total mass  $m_w$  of 1000 g on top of the cart. Record the total mass of the cart  $m_c = m_{ec} + m_w$  in Table 9.3.
21. Place the cart at its starting position from part A. Place just enough mass on the hanger to pull the cart at **constant speed** after you tap it to overcome static friction. Before you release the cart, press *Start* in Data Studio. When the cart strikes the bumper, press *Stop*. Use the graph in Data Studio to determine when the cart's speed is constant (when the speed is constant, the graph will be a horizontal line). If the hanger doesn't hit the floor before the cart hits the bumper, replace the string with a longer piece and remark the starting position of the cart.
22. Once you have achieved a constant speed for the cart, pull the cart to its start position and hold it in place. Use a meter stick to measure the height of the bottom of the mass

- hanger  $y_h$ . Record the total hanging mass  $m_h$  and the initial height of the hanger  $y_h$  in Table 9.3. Calculate the heat  $\Delta H$  generated by friction and record it in Table 9.3.
23. Perform four additional trials, increasing the inclination angle by  $5^\circ$  for each trial, for a total of five trials. Record your data in Table 9.3.
  24. Close Data Studio and switch off the computer system. Unplug, coil, and secure the sensor cable. Return all your equipment to the lab cart and clean up you lab table area.