UNIVERSITY PHYSICS

EXPERIMENT 7

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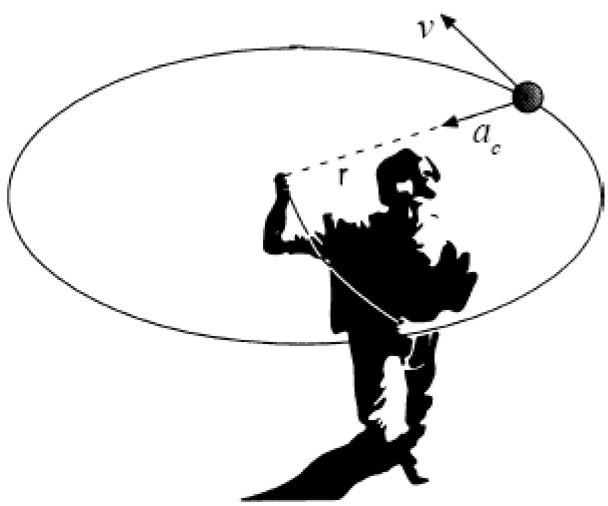
COLLIN COUNTY COMMUNITY COLLEGE

Centripetal Acceleration



Produced by the Physics Staff at
Collin County Community College

Purpose

The purpose of this activity is to experimentally investigate the relationships between centripetal acceleration, circular path radius, radial force, and object mass.

Equipment

- 1 Rotating Platform
- 1 300 g square mass for platform
- 1 Mass and Hanger Set
- Spool of thread
- Radial Acceleration Accessory
- 1 Pulley
- 1 Stopwatch

Introduction

In his first law of motion, Isaac Newton tells us that an object will continue in a state of rest or a state of motion at constant speed in a straight line (i.e., a state of constant velocity) unless a net external force compels it to change that state. The property that makes an object try to remain in its state of motion is its inertia. When we measure this property we call it *mass* rather than inertia.

If Newton's first law describes an object's state of motion in the absence of an applied force, his second law describes what happens to that state when an external force *is* applied. The first law implies that if you exert a net force on an object, its constant velocity will change. The second law tells us not the amount of change, Δv, but the *rate* of that change, dv/dt. The law states that the velocity changes at a rate that is directly proportional to the magnitude of the applied force and inversely proportional to the object's mass. What is more, the law reveals that the direction of the *change* in velocity is the same as the direction of the force.

Whenever you see an object moving in a curved path, you know that some force must be acting on it to cause it to deviate from a straight line path. This force must be directed normal to the path, i.e., radially toward the curved path's instantaneous center. The resulting rate of change in the object's velocity is its radial acceleration. If the path is circular, its instantaneous center is a fixed point. But if the path has varying curvature, its center moves as the radius varies. Either way, the object's radial acceleration, and of course the applied force, are always directed toward the center. Since the path itself is always tangential, the object's velocity and its radial acceleration are always normal to each other.

The force causing an object to move in a curved path is *not in addition* to other forces that may be acting on it. It is, rather, the radial component of the *vector sum* of all these forces. The forces are exerted on the object by some aspect of its surroundings; such as the sideways static frictional force by the road exerted on your car's tires as you turn a corner, or the earth's gravitational force exerted on an orbiting satellite (pulling it out of a straight path into a circular one), or the tension force in the string as you swing an object around your head.

Any object that is moving in a circular path of radius *r* and with constant tangential speed *v* has a radial acceleration given by:

$$a_c = \frac{v^2}{r}$$

directed toward the center of the circle. In the case of a boy swinging a ball of mass *m* around his head on a string, the radial force on the ball is:

$$F_c = ma_c$$

This force is supplied by the boy's hand and is transmitted to the ball through the tension force in the string.

In this experiment, you will examine the relationships between the properties of an object of mass *m* that is moving in uniform circular motion. To do this, you will measure the effects on the object's period of revolution *T* when you vary, one at a time, its mass, the radius of its path, and the radial force being exerted on it. This radial force F_c exerted on the object will be supplied by a stretched spring, and you will be able to directly measure it. You will also calculate the force from its relationship to the object's other properties: its mass *m*, the radius of its path *r*, and the period of its revolution *T*. You will compare the value of F_c that you calculate to the value you measure directly. While performing this experiment you will learn to:

1) identify the radially-directed force that causes any circular motion,
2) describe how the magnitude of this force may be calculated from the properties of the object in motion,
3) summarize the relationships between any of the properties of an object in uniform circular motion.

Theory

An object in uniform circular motion moves with a constant speed. But its velocity is varying even though its speed is constant because the *direction* of its motion is changing. That is, the velocity vector changes direction but not magnitude. Since both *v* and Δv are vectors, the only way you can have a non-zero Δv without changing the magnitude of *v* is for Δv and *v* to always be perpendicular, i.e., the object's acceleration is always perpendicular to its velocity – always directed toward the center of the circle.

The magnitude of this radial acceleration a_c is related to the object's tangential speed *v* and to the radius of its circular path *r*.

According to Newton's laws, any acceleration must be the result of a force being exerted on the object by something. The radial force required to move an object of mass *m* with a radial acceleration a_c is then:

$$F_c = ma_c = m\frac{v^2}{r} \qquad \textbf{\textit{Equation 7.1}}$$

You can verify this relationship by measuring the object's mass m and then measuring each of the other three properties, F_c, v, and r, while the object is moving in uniform circular motion. Although it is usually difficult to measure tangential speed along a circular path directly, you can readily *calculate* v by measuring the time for one rotation (the period T), which is inversely related to the speed. The speed of an object in uniform circular motion is defined as:

$$v = \frac{(\text{Circumference})}{(\text{Period})} = \frac{2\pi r}{T}$$ ***Equation 7.2***

This equation tells us that as long as r is constant, v and T are inversely proportional. You can then write the relationship between the radial force and the period as:

$$F_c = \frac{m(2\pi r)^2}{rT^2} = \frac{4\pi^2 rm}{T^2}$$ ***Equation 7.3***

As long as r and m are constant, F_c is inversely proportional to T^2. In other words, for constant r and m, if you double F_c, T will decrease by a factor of four – the mass will revolve four times faster. You can also express F_c in terms of the frequency of revolution f. Since the frequency is the reciprocal of the period ($f = 1/T$), the latter equation becomes:

$$F_c = 4\pi^2 rmf^2$$ ***Equation 7.4***

where f is measured in revolutions per second.

Procedure

You will use a rotational system consisting of the *Centripetal Acceleration Accessory* (shown in Figure 7.2) and the *Rotating Platform Assembly* (shown in Figure 7.3). Before you begin the experiment, assemble the system as follows:

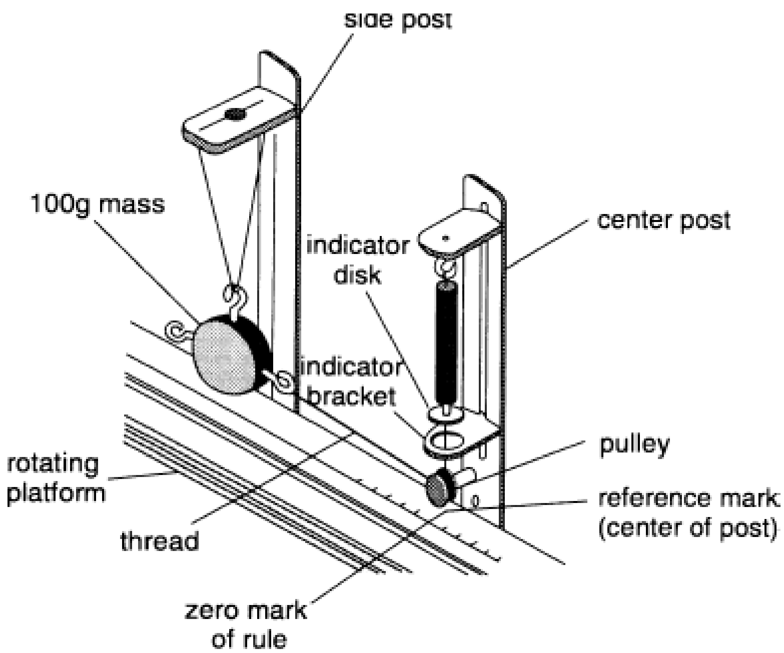


Figure 7.2. Radial Acceleration Accessory

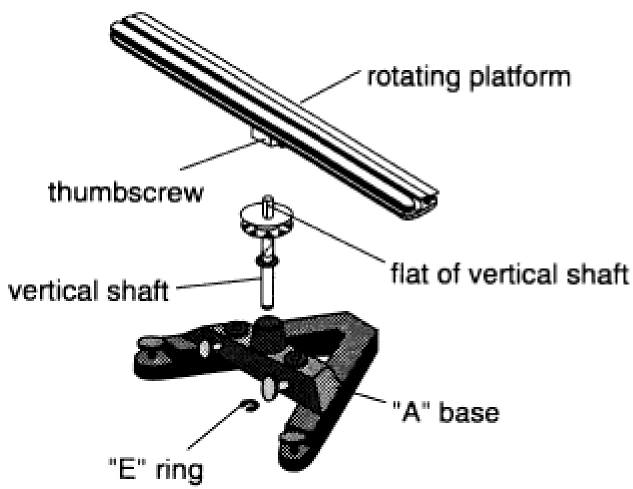


Figure 7.3. Rotating Platform Assembly

Center Post Assembly

If the center post assembly (with spring and indicator) has not already been assembled, the following instructions will guide you through the assembly process:

1) Hook the top of the spring to the spring bracket and connect the indicator disk to the lower end as in Figure 7.4. Insert the spring bracket into the top of the slot on the Center Post and tighten its thumb screw.

2) Cut a piece of thread 30 cm long. Tie one end to the bottom of the indicator disk and tie a loop in the other end. Insert the indicator bracket into the bottom of the slot on the Center Post and tighten its thumb screw.
3) Insert the pulley in the higher of the two holes near the bottom of the Center Post.
4) Insert the winged thumb screw in the hole at the bottom the Center Post and attach the square nut. Slide the square nut into the T-slot on the side of the platform that has the ruler.
5) Align the reference mark on the Center Post with the zero mark on the ruler and tighten the thumb screw.

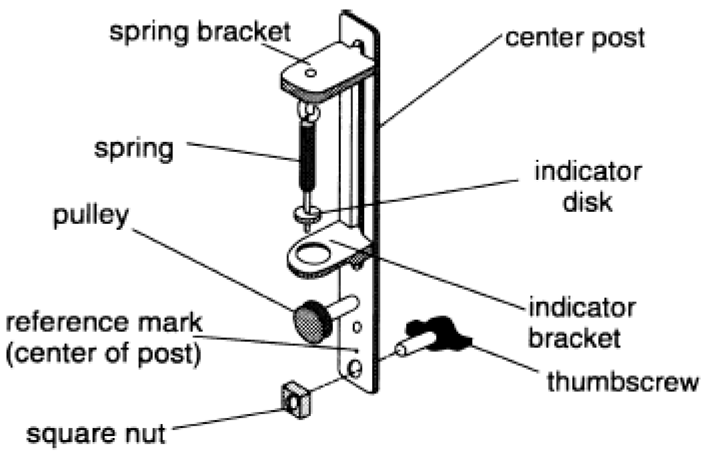


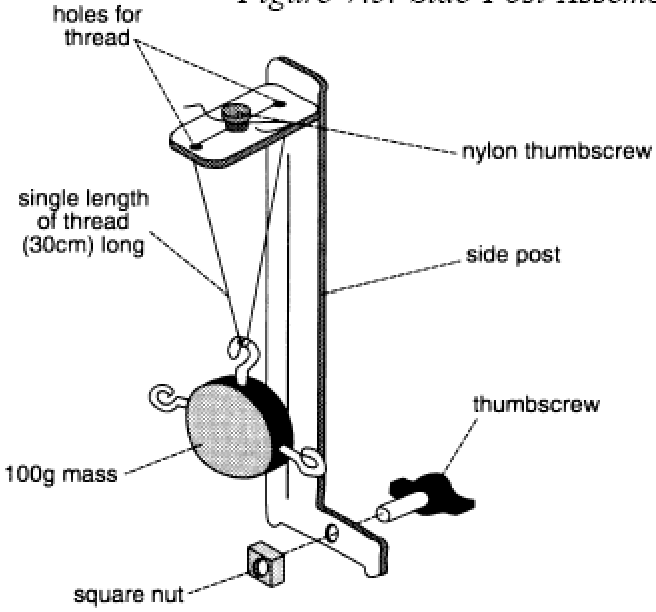
Figure 7.4. Center Post Assembly

Side Post Assembly

If the side post assembly (with revolving mass) has not already been assembled, the following instructions will guide you through the assembly process:

1) Cut a second length of thread about 30 cm long and tie one end around the nylon thumb screw on the arm at the top of the Side Post. Push the other end down through one of the holes in the arm and then back up though the other hole. Do *not* pull the thread taut.
2) Loosen the nylon thumb screw, wrap both ends of the thread around the screw, then tighten the screw, as in Figure 7.5.
3) Insert the winged thumb screw in the hole at the bottom of the Side Post and attach the square nut.
4) Slide the square nut into the T-slot on the same side of the platform as the Center Post. Slide the Side Post to align the index line with the 20-cm mark on the ruler and tighten the winged thumb screw.

Figure 7.5. Side Post Assembly



Leveling the Base

1) Attach the square black 300-gm mass onto the opposite end of the platform from the Side Post (see Figure 7.6). Its exact position is not critical. Tighten the screw so it will not slide.
2) Turn the right leveling screw on the A-frame base clockwise to raise the right foot of the base until the heavy end of the platform (the square mass) is over the left leveling screw, as in Figure 7.6 left.
3) Without moving the base, rotate the platform 90° so it is parallel to the right leg as in Figure 7.6 right, then turn the right leveling screw clockwise until the platform stays in this position without rotating on its own.
4) The base and platform are now level and the platform will remain at rest at any rotational orientation. **If you move the base you will have to repeat this procedure to re-level it.**

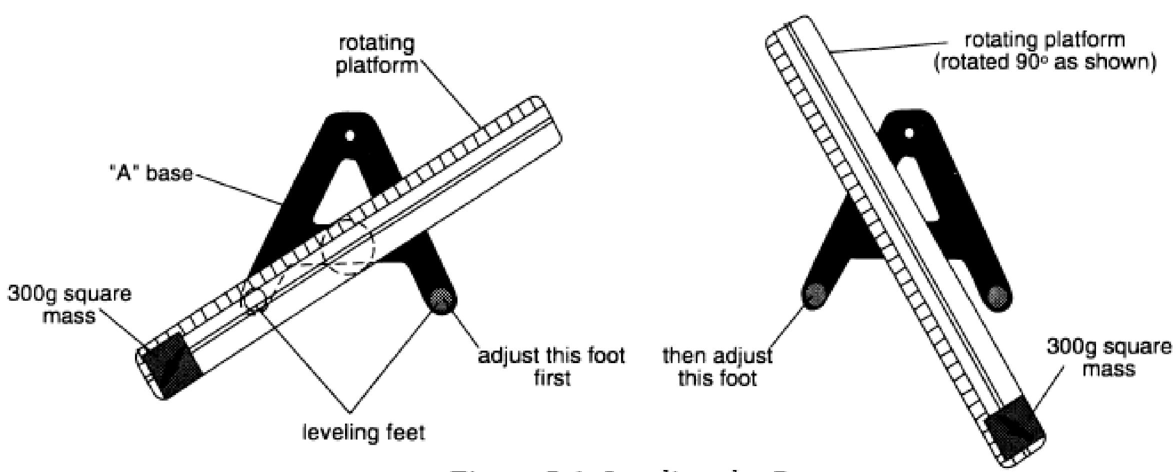


Figure 7.6. Leveling the Base

Threading the Centripetal Force Accessory

1) Hang the bare 3-hook disk (without additional disks attached) from its center hook from the thread loop on the Side Post as in Figure 7.2. This will be the revolving mass m_r.
2) Run the thread from the Center Post down through the large hole in the indicator bracket, around the pulley, and over to the right hook on the revolving mass m_r as in Figure 7.2. This will provide the radial force F_c necessary to keep m_r in a circular path of radius r as it revolves.
3) By loosening the nylon thumb screw on top of the Side Post arm, adjust the height of the revolving mass so the string from the Center Post pulley is parallel to the platform (horizontal). Measure its height at each end to be sure.

A: Variable Radius, Constant Force and Mass

The Center Post is mounted on the rotational axis of the platform. When the platform rotates, the Side Post will revolve around the axis, with the mass m_r hanging from it swinging away from the axis as it does. To make the mass revolve at the radius r, it must be pulled inward (toward the axis). The radial force F_c required to pull m_r into the path of radius r will be provided by the thread and the spring on the Center Post.

1) Unhook the revolving mass m_r from the thread loop on the Side Post, weigh it, record the mass in Table 7.1, then re-hang it from the Side Post arm. Measure the radius r of the revolving mass' path ***when it is hanging vertically***, and record this value in Table 7.1. Reattach the string from the center post. The center post string should be short enough such that the revolving mass is pulled toward the center post by the spring.
2) Attach the clamp-on pulley to the end of the platform beyond the Side Post as in Figure 7.7. Cut a third piece of thread about 30 cm long and tie loops in both ends. Hook one end to the revolving mass, run it over the pulley, and hook the other end to the mass hanger. Place enough mass on the hanger to pull the revolving mass to its ***vertical position***, and record the *total* hanging mass m_h (including the hanger) in Table 7.1 (when m_r is hanging vertically, the thread loop it is hanging from will be aligned with the index line on the inside of the Side Post).
3) Adjust the lower (indicator) bracket on the Center Post (Figure 7.2), so that the orange indicator is centered vertically in the large hole in the bracket. The weight w hanging from the pulley is now being balanced by the spring force as in Figure 7.7. Record this force in Table 7.1. When the revolving mass m_r is aligned with the index line, the spring will be exerting a radial force $F_c = w$ on it. This will be your *measured* value of F_c.
4) Before you rotate the platform, you must unhook the third thread, remove the weight hanging from the pulley, and ***remove the pulley***. This will cause the spring to pull the mass m_r inward (toward the axis) and will raise will the orange indicator above the indicator bracket.
5) To find the angular speed or period of the rotating platform, rotate the platform by twirling the knurled portion of the shaft between your fingers. Increase its rotational speed until the orange indicator is vertically centered in the indicator bracket on the Center Post. This tells you that the revolving mass m_r is once again

hanging vertically and is thus revolving at the desired radius r. Practice maintaining the exact speed required to keep the indicator centered.

6) While one team member maintains this rotational speed, another member will use the stopwatch to time ***ten full revolutions*** of the rotating platform. Record this time t in Table 7.1. The period T is then $t/10$.
7) Repeat steps 2–6 for four additional radii. Record the values of each new radius r and period T in Table 7.1. For each radii, ***don't move the indicator bracket!*** In order to maintain a constant radial force, change the radius of motion by moving the side post and changing the length of the string.
8) Calculate the value of T^2 for each of the five trials, recording it in Data Table 7.1, and plot T^2 vs. r.
9) Measure and record the slope of the graph. Using Equation 7.3, calculate the value of F_c from the value of the slope and record it in Data Table 7.1. This will be your *calculated* value of F_c.
10) Calculate and record the percent difference between your *measured* and *calculated* values of F_c.

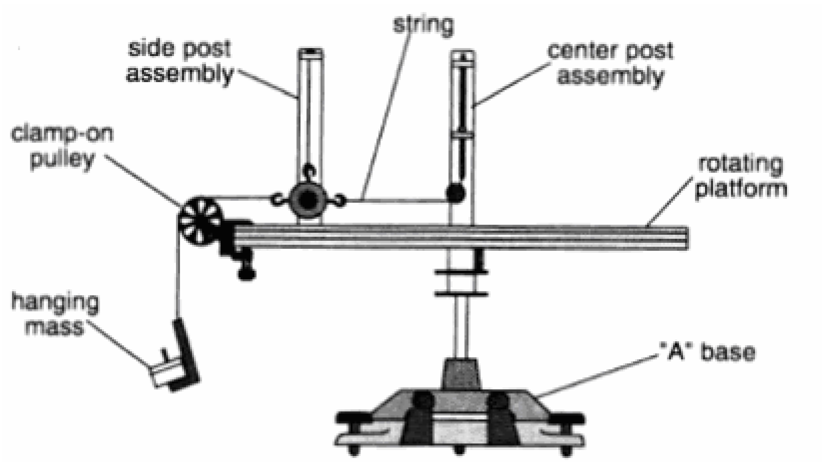


Figure 7.7. Measuring F_r

B: Variable Force, Constant Radius and Mass

11) Repeat steps 2–6 in Part A, except keep the radius r constant at 20 cm and vary the radial force F_c on the revolving mass in five more trials. For each force, ***don't move the side post!*** To change the force, change the length of the string without moving the side post. Then, with the revolving mass hanging vertical, move the indicator bracket to the new location of the indicator disk. Record all your measurements of hanging mass m_h, radial force F_c, and period T in Table 7.2.
12) Calculate the reciprocal of T^2, record it in Table 7.2, and plot a graph of T^{-2} *vs.* F_c.
13) Measure and record the slope of the graph, then calculate the value of m_r from the value of the slope and record it in Table 7.2.
14) Calculate and record the percent difference between your measured and calculated values of m_r.

C: Variable Mass, Constant Radius and Force

15) Add/remove one of the two additional side masses on the revolving mass and weigh it again. Record this new value of m_r in Table 7.3.
16) Repeat steps 2–6 in Part A, except keep F_c and r constant and vary m. Do one trial with one side disk attached to the revolving mass, one trial with both side disks attached, and one trial with no side disks attached. Record all your measurements of m_r, and T in Table 7.3.
17) Calculate the value of T^2 for each of the three trials, recording it in Data Table 7.3, and plot a graph of T^2 vs. m_r. From the slope of this graph, calculate the value of F_c and record it in Table 7.3.
18) Calculate and record the percent difference between measured and calculated values of F_c.
19) Return all your equipment to the lab cart and clean up your lab table area.