UNIVERSITY PHYSICS

EXPERIMENT 4

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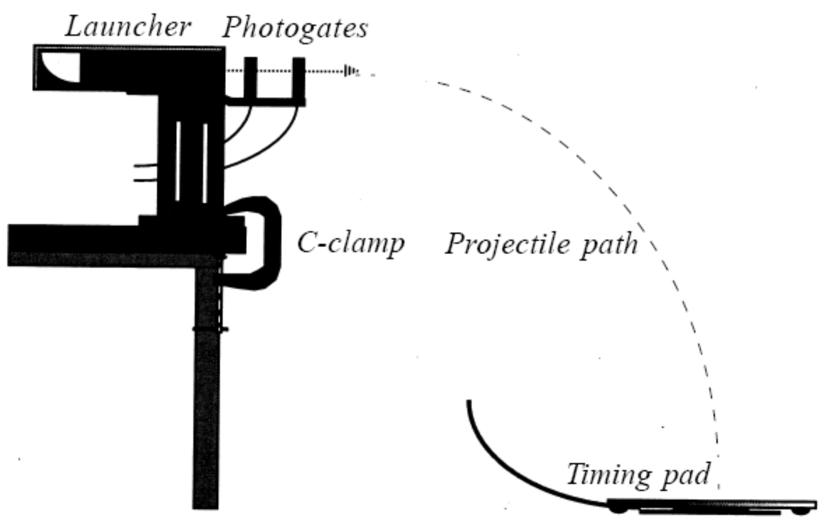
COLLIN COUNTY COMMUNITY COLLEGE

Two-Dimensional Motion



Produced by the Physics Staff at
Collin County Community College

Purpose

In Experiment 2 you explored the motion of an object when its displacement, velocity, and acceleration vectors were all in the same direction. In this experiment, you will investigate the motion that results when these vectors can have any direction in a plane, i.e., in two-dimensional space.

Such motion is far more common in nature than motion in one dimension; indeed, it is even more common than motion in three dimensions. Examples of natural and man-made projectile motion range from thrown balls to fired bullets and arrows to the stream of electrons in a TV picture tube.

Equipment

- 1 Projectile Launcher w/ Rod
- 2 Photogates w/ Bracket
- 1 Time-of-Flight Pad
- 1 Pasco Extension Cable
- 1 C Clamp
- 1 Stainless Steel Ball
- Meter Sticks

Introduction

It is obvious that any measurable quantity has a numerical value or magnitude. When you measure such a quantity, its magnitude tells you the number of units contained in that measurement. Some measurable quantities, such as time, volume, mass, or temperature, are completely specified by their magnitude – no other descriptors are necessary. These are called scalar quantities or, simply, scalars.

In your lecture class, you learned that you can determine the position $\mathbf{r}(t)$ and velocity $\mathbf{v}(t)$ at any time t of an object moving with constant acceleration if you know (or can measure) its original position $\mathbf{r}_o$, its original velocity $\mathbf{v}_o$, and its acceleration $\mathbf{a}$. *As long as its acceleration is constant*, the motion of the object is completely described by the following two relationships, called its equations of motion:

$$\mathbf{r}(t) = \mathbf{r}_o + \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2$$

and

$$\mathbf{v}(t) = \mathbf{v}_o + \mathbf{a}t$$

Since these are vector equations, the resulting position and velocity of the object depend on both the magnitude and direction of its initial velocity $\mathbf{v}_o$ and of its constant acceleration $\mathbf{a}$. You saw in Experiment 3 that when $\mathbf{v}_o$ and $\mathbf{a}$ are both in the same direction, the resulting motion is one-dimensional. You will now investigate the motion when $\mathbf{v}_o$ and $\mathbf{a}$ are in different directions.

Theory

Projectile Motion

The motion of a projectile is a special case of a freely falling body: **a** is downward and constant but $\mathbf{v}_o$ is in some direction other than vertical. The projectile's path [the locus of points defined by **r**(*t*)] is a parabolic curve produced by the initial velocity $\mathbf{v}_o$ and the downward acceleration of gravity **g**, as in Figure 4.1.

You can study this type of motion most effectively if you think of it as the sum of of two independent motions: a horizontal (*x*) component with constant velocity ($\mathbf{a}_x = 0$) and a vertical (*y*) component with velocity changing at a constant rate ($\mathbf{a}_y = -\mathbf{g}$). As usual, neglecting air drag simplifies the problem.

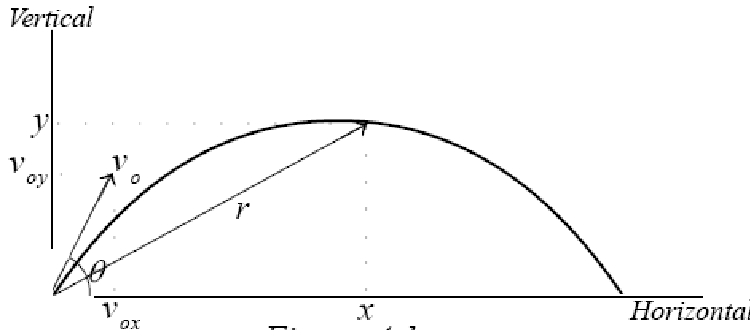


Figure 4.1

1. Horizontal component of motion

Since $\mathbf{a}_x = 0$, $\mathbf{v}_x$ remains constant at whatever its initial value was. Thus, the object's horizontal component of motion is fully described by the relationship for one-dimensional motion at constant velocity

$$\mathbf{x}(t) = \mathbf{v}_{xo}t = (\mathbf{v}_o \cos\theta)t$$

where $\mathbf{v}_o$ is the object's initial velocity and θ is the angle between the horizontal and $\mathbf{v}_o$. The horizontal distance **x**(*t*) traveled by the object is called its *range*. As the equation above reveals, the value of **x**(*t*) depends on $\mathbf{v}_{xo}$ and *t* (its time of flight). The flight time depends on the *vertical* distance it moves before its flight ends. Obviously, the sooner the flight ends, the shorter flight time and, hence, the shorter the range. A projectile launched from the table top to the floor falls a greater vertical distance than the same object launched under the same conditions from table top to table top, so both its flight time and range will be greater.

2. Vertical component of motion

Since $\mathbf{a}_y = -\mathbf{g}$, the vertical velocity $\mathbf{v}_y$ increases in the downward direction. The object's vertical component of motion is therefore described by the one-dimensional equations of motion at constant acceleration:

$$\mathbf{y}(t) = \mathbf{v}_{yo} t - \frac{1}{2}\mathbf{g}t^2$$

and $\mathbf{v}_y(t) = \mathbf{v}_{yo} - \mathbf{g}t$

where $\mathbf{v}_{yo} = \mathbf{v}_o \sin\theta$ is the vertical component of $\mathbf{v}_o$, and $\mathbf{v}_y(t)$ is the vertical component of the object's instantaneous velocity. If $\mathbf{v}_{yo}$ is downward, $\mathbf{v}_y(t)$ will increase continuously. If $\mathbf{v}_{yo}$ is upward, $\mathbf{v}_y(t)$ will first decrease to zero then increase continuously in the downward direction.

3. Time of Flight for horizontal launch

Since the object's vertical motion is independent of its horizontal motion, the time it takes to fall to the floor depends only on the vertical distance it falls $\mathbf{y}_o$ and on the vertical component of its initial velocity $\mathbf{v}_{yo}$. It is independent of its horizontal velocity. If the object is launched horizontally, i.e., if $\mathbf{v}_{yo} = 0$, then $\mathbf{y}(t) = \frac{1}{2}\mathbf{g}t^2$ and $t = \frac{2y_o}{g}$, the same amount of time as that taken by an object dropped from rest. Since t does not depend on $\mathbf{v}_o$, this relationship gives the time of flight for any projectile launched horizontally regardless of its initial speed.

4. Time of Flight for non-horizontal launch

If the object is launched at an upward angle θ, its initial velocity will have a horizontal component magnitude of

$$v_{xo} = v_o \cos\theta$$

and a vertical component magnitude of

$$v_{yo} = v_o \sin\theta$$

Its time of flight will be longer because it has to rise a vertical distance

$$y_u = \frac{v_{yo}^2}{2g},$$

then start falling from rest from that height. The time it takes to rise to that height is

$$t_u = \frac{v_{yo}}{g}$$

Since it then falls from rest from that height, the time for its downward flight is

$$t_d = \sqrt{\frac{2y_d}{g}}$$

where y_d is the total height it falls. Its total time of flight is therefore

$$t_T = t_u + t_d = \frac{v_{yo}}{g} + \sqrt{\frac{2y_d}{g}}$$

If the object is launched at a downward angle, its time of flight will be shorter because it begins its fall with an initial downward component of velocity. To calculate the time of flight from a given height y_o with a given initial downward component of velocity v_{yo}, you must solve the quadratic relationship

$$-y_o = -v_{yo}t - \frac{1}{2}gt^2$$

for time. To do this, first rewrite the above equation in standard quadratic form ($ax^2 + bx + c = 0$). Then use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to solve for the variable t:

$$t = \frac{v_{yo} \pm \sqrt{v_{yo}^2 - 4\left(-\frac{g}{2}\right)(y_o)}}{-g} = \frac{-v_{yo} \mu \sqrt{v_{yo}^2 + 2gy_o}}{g}$$

The positive root of the radical is the only one that has physical meaning (using the negative root results in a negative time).

A quick calculation using typical values reveals that the flight time is 0.64 sec when v_{yo} is 0 m/s and y_o = 2 m, i.e., when the object falls from rest from a height of 2 m.

5. Range of Projectile

Since the horizontal component of its velocity is constant, the horizontal distance (range) of the projectile's flight is given by the equation: $\mathbf{x}(t) = \mathbf{v}_{xo}t = (\mathbf{v}_o \cos\theta)t$. The time of flight is t_T, so the range can be expressed in terms of the initial values of height, speed, and angle:

$$x = v_o \cos\theta\left(\frac{v_o \sin\theta}{g} + \sqrt{\frac{2y_o}{g}}\right)$$

For a horizontal launch, θ is zero, so this equation becomes

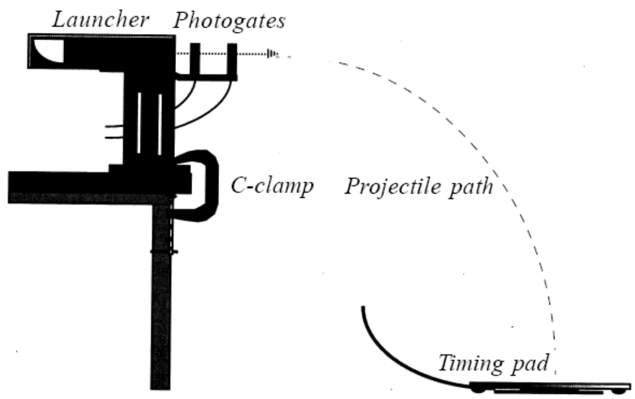
$$x = v_o\sqrt{\frac{2y_o}{g}}$$

You will verify these relationships between range, initial speed, and time of flight in this experiment.

Procedure

You will examine 2-D projectile motion by measuring the path and time of flight of a small projectile shot from a spring gun. With the power switcher off, connect the following sensors to the Interface:

1. Plug two photogates into Digital Channels 1 and 2.
2. Plug the Timing Pad, through its extension cord, into Digital Channel 3.



You will use two photogates to measure the initial speed of the ball as it comes out of the launcher. You will then use the first photogate and the pressure pad to measure the ball's time of flight. Data Studio records and displays both the initial speed and the time of flight.

1. Clamp the base of the projectile launcher to the edge of the table as shown in the figure above. Aim the launcher away from the table toward the center of an open area of at least 3 meters. Adjust the angle of the launcher to 0° so the steel ball will be launched horizontally.
2. Slide the photogate mounting bracket into the T-slot under the launcher muzzle. Mount the first photogate in the bracket position closest to the muzzle of the launcher. Mount the second photogate in the position farthest from the launcher muzzle.
3. Measure the center-to-center distance between the two photogate's lenses. Record the measurement in meters in the designated place above Tables 4.1 and 4.2.
4. Switch on the computer system, open Data Studio, and select Create Experiment. Double-click the Photogate sensor icon twice and the Time of Flight sensor icon once to show two photogates and the time-of-flight accessory plugged into digital channels 1, 2, and 3 on the interface.
5. Click the *Timers* button. Under Timing Sequence Choices, select *Blocked* for Ch 1 and *Blocked* for Ch 2. Type *Time between gates* as the label. Click the *New* button and select *Blocked* for Ch 1 and *On* for Ch 3. Type *Time of flight* as the label. Click *Done.*
6. Double click on Digits in the Displays listing, select *Time between gates*, and click OK. Double click on Digits again, select *Time of flight*, and click OK. Drag the two Digits displays below the Experiment Setup window.

1. Horizontal Launch Angle

7. Measure the vertical height of the launcher muzzle (a small set of crosshairs printed on the side of the launcher mark the exit point of the ball) above the floor. Record the height in meters in the designated place above Table 4.1.

8. Use the plumb bob to locate the point on the floor that is directly beneath the launcher muzzle. Stick a piece of tape on the floor and mark the point with a dot on the tape.
9. Put a steel ball into the launcher and, using the ramrod, cock it to the **short range** position.
10. Fire the launcher a couple of times and place the pad where the ball hits the floor. ***Be sure nobody is standing in the flight path of the ball before launching!***
11. Reload the ball into the launcher and cock it again to the short range position.
12. Click the Start button and launch the ball. Note the ball's impact point on the pad, then click the *Stop* button to halt data recording. This allows you to reload the launcher without accidentally recording data when you block the photogates.
13. Measure the range of the ball and record it, the time between gates, and the time of flight in Table 4.1.
14. Repeat steps 9–13 but with the launcher cocked to the **middle range** position.
15. Repeat steps 9–13 but with the launcher cocked to the **long range** position.
16. Using the measured time between gates and launch height of the ball, calculate the launch velocity, time of flight, and range for each launch speed and record them in Table 4.1.
17. Calculate and record in Table 4.1 the percent differences between the measured and calculated flight times and ranges.

2. Non-horizontal Launch Angle

18. Adjust the angle of the launcher to 30° above horizontal and cock it to the short range position. Test-launch the ball a couple of times before placing the pressure pad.
19. Measure the vertical height of the launcher muzzle above the floor. Record the height in the designated place above Table 4.2.
20. Use the plumb bob to locate the point on the floor that is directly beneath the launcher muzzle. Stick a piece of tape on the floor and mark the point with a dot on the tape.
21. When you are ready, click the Start button and launch the ball at the 30° angle. Note the ball's impact point on the pad. Don't forget to click Stop.
22. Measure the range of the ball, and record the range, time between gates, and time of flight in Table 4.2.
23. Repeat steps 21 and 22 but with the launcher cocked to the middle range position.
24. Repeat steps 21 and 22 but with the launcher cocked to the long range position.
25. Using the measured time between gates and launch height of the ball, calculate the launch velocity, time of flight, and range for each launch speed and record them in Table 4.2.
26. Calculate and record in Table 4.2 the percent differences between the measured and calculated flight times and ranges.