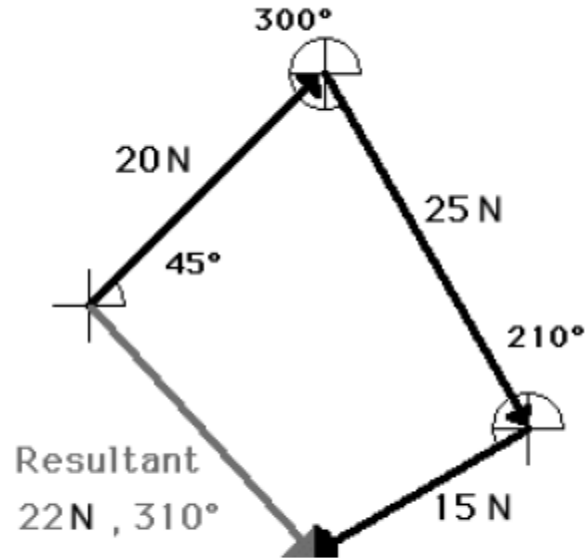


Resolving & Adding Vectors



*Produced by the Physics Staff at
Collin County Community College*

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Purpose

You will resolve the three force vectors below into their x and y components by all three methods (graphical, analytical, and experimental), and compare the results. Then you will add them using all three methods.

Equipment

- 1 Force table set with 4 pulleys and 1 white ring
- 1 Set of masses with four hangers
- 1 Level
- 1 Ruler
- 1 Small protractor
- 1 Roll of string

Introduction

It is obvious that any measurable quantity has a numerical value or magnitude. When you measure such a quantity, its magnitude tells you the number of units contained in that measurement. Some measurable quantities, such as time, volume, mass, or temperature, are completely specified by their magnitude – no other descriptors are necessary. These are called scalar quantities or, simply, scalars.

Other quantities, such as displacement, velocity, acceleration, or force, require something more than just their magnitudes. To describe them completely, you must measure and record their direction as well as their size. These are called vector quantities or vectors.

We distinguish between these two types of measurable quantities because we have to use different techniques when we add or subtract vectors than we do when we add or subtract scalars. The simple rules of addition and subtraction that you learned in grade school apply only to scalars. If you want to determine the total mass of a group of individual objects, or the total time of a series of time intervals, you simply add the individual values.

But if you want to determine the net effect of several different forces acting on an object, or of several displacements from a starting point, you cannot simply add the individual forces or displacements unless they are all in the same direction. In the more general case, you must consider the direction of each quantity.

There are three ways you can take the directions of vectors into account when you add them; and all three ways are included in the general term vector addition. Analytic methods of vector addition (those using only equations) employ trigonometry and algebra. Graphical methods employ trigonometry and geometry. Experimental methods use equipment and measuring devices.

All three methods of vector addition frequently use a technique called resolving the vector in which you replace a given vector by two new vectors called its components.

The components of a vector are always orthogonal (perpendicular to each other) because they lie in the directions of the axes of a coordinate system. They are called the x component and the y component of the original vector, and their magnitudes are such that their vector sum is equal to the original vector.

To resolve a vector in some arbitrary coordinate system means to replace it with its two components. The directions of the components depend, of course, on the directions of the coordinate system's axes. A given vector quantity in different coordinate systems will have different components, but the vector sum of the two components in any coordinate system will always be equal to the given vector (both magnitude and direction).

Resolving a vector into its components allows you to treat each component as if it were a scalar, making calculations easier. Since the x components of all vectors in a given coordinate system are all parallel to the x axis, you can add or subtract them using scalar arithmetic.

In this experiment, you will use and compare graphical, analytical, and experimental methods to explore the concepts of

- a) resolving a vector into, and representing it by, its orthogonal components
- b) adding vectors to determine their sum (also called their resultant)

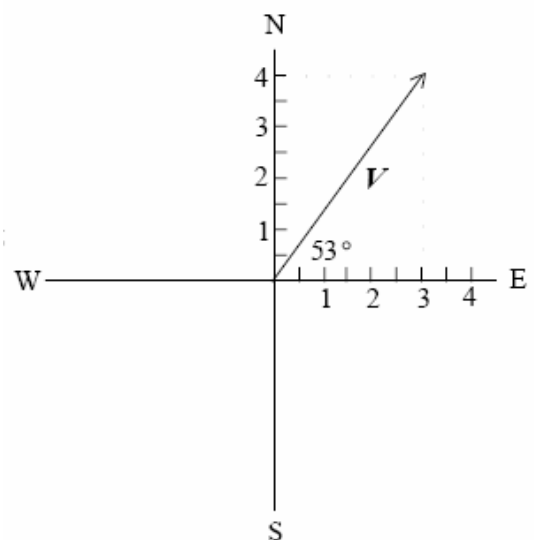
In the experimental part, you will use forces applied to an object as specific examples of vectors.

Vector Representation

A vector can be represented graphically by an arrow, drawn to some scale, whose length represents the magnitude of the vector and whose orientation in space represents the direction of the vector.

For example, suppose that you wished to draw a velocity vector \mathbf{V} of 50 mph in a direction 53° north of east. First, you would draw a pair of orthogonal axes in the four compass directions. Then you would draw a line through the origin that is 53° counter clockwise from the east axis, as in the figure shown.

To determine the length of the arrow, you use trigonometry. Note that $\tan 53^\circ = 1.33 = 4/3$, so \mathbf{V} must be the hypotenuse of a 3-4-5 right triangle in which the 3 leg parallels the east axis and the 4 leg parallels the north axis. You would therefore choose a scale of 1 unit = 10 mph, so the length of \mathbf{V} would be 5 units (50 mph).



In computer-printed text, you signify a vector quantity by printing its algebraic symbol in **boldface** type. In handwritten text, you draw an arrow over the symbol. For example, you would print the algebraic symbol for a force vector as **F** and you would write it as \vec{F} .

*Note: An arrow representing a vector quantity has magnitude and direction, just like the quantity it represents, but it does **not** have a unique position. You can place the arrow anywhere without changing its vector characteristics (its magnitude and direction).*

Theory

A. Vector Resolution

Any vector quantity can be resolved into its components by three different methods: graphical, analytical, or experimental.

A1. Graphical resolution

The components of a vector are defined as the projections of the vector onto two arbitrary axes. It is common to use axes in the horizontal (x axis) and the vertical (y axis) directions. So if vector **V** in the figure above makes an angle θ with the x axis, it has components V_x along the x axis and V_y along the y axis. The length of V_x is obtained graphically by dropping a line from the tip of vector **V** normal to the x axis. V_x is the distance from the origin to the normal line. Similarly, the length of V_y is obtained graphically by dropping the line from the tip of **V** normal to the y axis so V_y is the distance from the origin to that normal line.

A2. Analytical resolution

Analytically, the magnitudes of V_x and V_y can be found by using the trigonometry of a right triangle. The vector **V** and its two components V_x and V_y always form a right triangle whose hypotenuse is **V** and whose sides are V_x and V_y . You then find the components V_x and V_y by trigonometry:

$$V_x = V \cos \theta$$

$$V_y = V \sin \theta$$

where V is the magnitude of the vector **V** and θ is the angle between the vector and the $+x$ axis.

Note: The x and y components of a vector have algebraic signs (+ or -) that depend on which quadrant the vector lies in ($\cos \theta$ is positive in the first and fourth quadrants, $\sin \theta$ is positive in the first and second).

A3. Experimental resolution

You can resolve force vectors experimentally on a horizontal circular table with pulleys that can be clamped at any position around its edge. The outer rim of this force table is calibrated in degrees from 0° to 360° . Strings are tied to a ring at the center of the table. Each string passes over a pulley and has a mass hanger hanging from its lower end. You can thus apply several horizontal forces to the ring by clamping the pulleys at appropriate angles and placing various masses on the hangers. A removable pin at the table's center prevents the ring from moving too far when the forces are unbalanced. The ring is stationary at the center when all the forces on it are balanced.

Assume that the origin of the coordinate system is located at the pin and the axes pass through $0^\circ - 180^\circ$ and $90^\circ - 270^\circ$. You determine the direction of the forces applied to the ring by clamping a pulley at the appropriate position on the circular scale. The magnitude of the force is equal to the total weight hanging on that string (hanger plus added masses). The total force is related to the total mass by

$$F = W = mg$$

where W is the total weight suspended from the string, including the weight of the hanger, m is the total mass suspended from the string, and g is the acceleration due to gravity.

The unit of mass you will use in this experiment is the kilogram (kg). In the metric system of units, the unit of force is the Newton (N) which is related to the basic units by

$$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}.$$

The average gravitational acceleration at the earth's surface, in metric units, is $g = 9.80 \text{ m/s}^2$. Therefore, the weight of an object (in N) is equal to the product of its mass in kilograms times 9.80 m/s^2 .

You can apply a single given force \mathbf{F} (of known magnitude and direction) to the ring by hanging the appropriate mass from a properly located pulley. The ring will move in the direction of \mathbf{F} . However, you can apply a second force \mathbf{E} to the ring so that the ring does not move. In this event, the ring is said to be in *equilibrium*. The balancing force \mathbf{E} is called the *equilibrant force*. The magnitude of \mathbf{E} is equal to the magnitude of \mathbf{F} , but the direction of \mathbf{E} is 180° from the direction of \mathbf{F} .

Experimentally, rather than apply a single equilibrant force to the ring, you can apply two perpendicular forces to it in such a way that the ring is in equilibrium. The two perpendicular forces are the equilibrants of F_x and F_y – their magnitudes are equal to the magnitudes of F_x and F_y but their signs are opposite. Since the magnitude of \mathbf{E} is equal to the magnitude of \mathbf{F} , the magnitudes of these two perpendicular balancing forces are equal to the magnitudes of the experimentally-determined x and y components of \mathbf{F} .

Since \mathbf{E} is 180° from \mathbf{F} , the two perpendicular balancing forces lie in the quadrants opposite to F_x and F_y . For example, on the force table, assume that the positive x axis is labeled 0° , the

positive y axis is 90° , the negative x axis is 180° , and the negative y axis is 270° . If the vector \mathbf{F} lies in the first quadrant of this coordinate system, its x and y components will lie on the 0° and 90° axes. However, the equilibrant vector \mathbf{E} will lie in the third quadrant, between 180° and 270° . The x component of \mathbf{E} will lie on the 180° axis and its y component will lie on the 270° axis.

B. Vector Addition

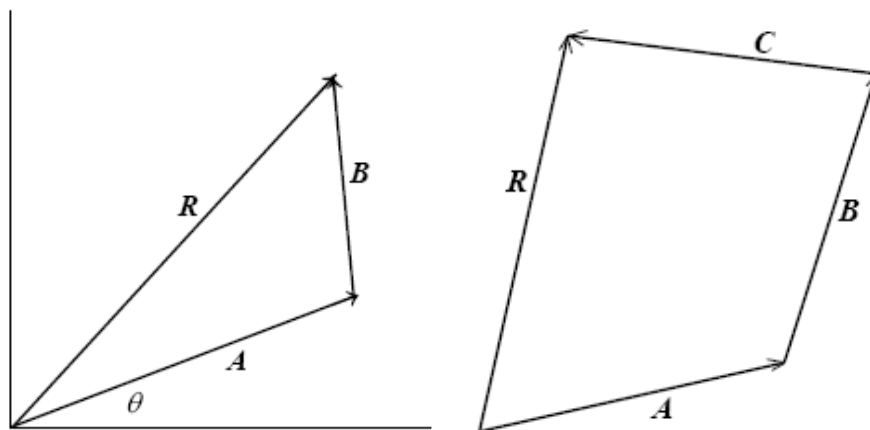
The sum of two or more vector quantities of the same kind (both forces, both velocities, etc.) is called the resultant \mathbf{R} of the vectors. The addition of two force vectors, \mathbf{A} and \mathbf{B} , can be written symbolically as

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The resultant of vector quantities is itself a vector quantity since it has both magnitude and direction. There are three methods for finding the resultant (i.e., the vector sum) of two or more vectors.

B1. Graphical method

You can graphically find the resultant of two arbitrary vectors \mathbf{A} and \mathbf{B} by drawing the vectors to some arbitrary scale and then moving \mathbf{B} (maintaining its length and direction) so that the tail end of \mathbf{B} is at the head end of \mathbf{A} . The resultant \mathbf{R} of the two vectors is then the vector you draw from the tail end of \mathbf{A} (the origin) to the head end of \mathbf{B} as in the top figure on the next page. You find the magnitude of \mathbf{R} by measuring its length on the diagram and comparing this length with the scale along the axes. You find the direction of \mathbf{R} with respect to the x axis by measuring the angle θ with a protractor.



You can use this procedure with any number of vectors. Place the tail end of each vector at the head end of the previous one, keeping their length and direction unchanged, then draw the vector \mathbf{R} from the tail end of the first vector to the head end of the last vector.

Suppose you want to add three vectors \mathbf{A} , \mathbf{B} , and \mathbf{C} . You can find the resultant of these three vectors graphically as shown in the lower figure at right. No matter where \mathbf{A} , \mathbf{B} , and \mathbf{C} were

originally located, you can move them to the positions shown without changing their magnitudes and directions.

B2. Analytical method

The graphical method for adding vectors shown in the lower figure may not be sufficiently accurate for the problem you are trying to solve. You have to measure \mathbf{R} 's length with dividers or a ruler and measure its direction with a protractor, and such measurements inherently have low precision and low accuracy. The analytical method of vector addition is a more accurate way of finding the sum of vectors. Using it, you can find the exact value of the resultant (both magnitude and direction) with the same numerical precision as the given vectors. The method gives exact equations that will enable you to find the magnitude and direction of the resultant of any number of vectors. To understand this method, first review the concept of vector components introduced in Part A.

Assume that you want to find analytically the sum of three vectors; that is, knowing \mathbf{A} , \mathbf{B} , and \mathbf{C} , you want to find \mathbf{R} , where

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C}$$

To find \mathbf{R} using the analytical method you would:

1. Find the x and y components of each given vector (A_x and A_y , B_x and B_y , C_x and C_y).
2. Find the x component of \mathbf{R} by adding the x components of all the vectors (using ordinary algebra since the x component vectors are all in the same direction):

$$R_x = A_x + B_x + C_x$$

3. Find the y component of \mathbf{R} by adding the y components of all the vectors:

$$R_y = A_y + B_y + C_y$$

4. As can be seen from the Pythagorean theorem, the magnitude of \mathbf{R} is given by

$$R = \sqrt{R_x^2 + R_y^2}$$

5. The direction of \mathbf{R} is given by the angle θ which is the angle, measured in a counterclockwise direction, between \mathbf{R} and the x axis. Thus

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

B3. Experimental method

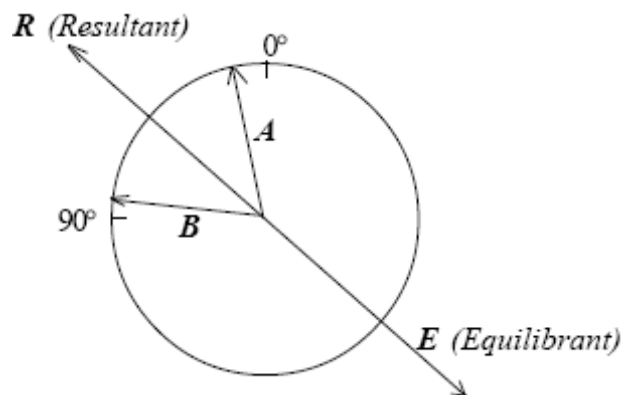
Vector quantities of any type can be added by either the graphical or the analytical method, but forces are the only type of vector that can also be added experimentally by use of a force table.

You can simultaneously apply several known forces (known magnitude and direction) \mathbf{A} , \mathbf{B} , and \mathbf{C} to the ring by hanging the appropriate mass from a properly located pulley. The ring

will move since there is a net force \mathbf{R} acting on it. But you can add a single additional force \mathbf{E} so that it does not move. It is then in equilibrium.

The resultant \mathbf{R} of the applied forces (\mathbf{A} , \mathbf{B} , and \mathbf{C}) acting on the ring is the vector that is equal in magnitude to \mathbf{E} but opposite in direction from it (i.e., \mathbf{R} is 180° from \mathbf{E}).

For example, the relationship between the resultant \mathbf{R} of two vectors \mathbf{A} and \mathbf{B} and their equilibrant \mathbf{E} is shown in the figure below. Notice that the length of the arrow representing \mathbf{E} is equal to the length of the arrow representing \mathbf{R} . However, the direction of the arrow representing \mathbf{E} is opposite to the arrow representing \mathbf{R} (that is, the equilibrant arrow is 180° from the resultant arrow).



Procedure

Data Studio is not needed for this experiment. Use the following values of mass and direction to produce the forces \mathbf{A} , \mathbf{B} , and \mathbf{C} unless told otherwise by your instructor:

Mass to be hung from each pulley and Direction with respect to the x axis:

$\mathbf{A} \rightarrow 50 \text{ g @ } 40^\circ$

$\mathbf{B} \rightarrow 70 \text{ g @ } 120^\circ$

$\mathbf{C} \rightarrow 100 \text{ g @ } 230^\circ$

For each of these forces, the total mass includes the mass hanger. Convert the total mass from grams to kilograms, and record the total mass hanging from each string in Data Table 3.1. Calculate the magnitudes of the forces (in N) and record the vector forces in Data Table 3.1.

A. Components of a Vector

A1. Graphical resolution

1. Using graph paper and a straightedge, draw x and y axes. Label the origin as point O and label both axes. Choose a scale (such as $1 \text{ in} = 2 \text{ N}$) which you will use to draw the vectors.
2. Draw vector \mathbf{A} using a protractor and ruler. Label the tip of \mathbf{A} as point E . Drop a line from the tip of \mathbf{A} normal to the x axis (don't just sketch it, set the straightedge parallel

to the graph paper's vertical lines). Label the intersection with the x axis as point D . The length OD represents the x component of vector \mathbf{A} . Use the x -axis scale to measure the length OD which is the magnitude of A_x (in N). Record the value of A_x in Table 3.2. The vertical length DE represents the y component of \mathbf{A} . Use the y -axis scale to measure the length DE which is the magnitude of A_y (in N). Record the value of A_y in Table 3.2.

3. Assuming the component values obtained by the analytical method in Part A2 to be the true values, calculate the percent error in your graphical values of A_x and A_y . Record the average value of the two percent errors in Table 3.2.
4. Repeat steps 1 – 3 for force \mathbf{B} on a separate sheet of graph paper and Table 3.2.
5. Repeat steps 1 – 3 for force \mathbf{C} on a separate sheet of graph paper and Table 3.2.

A2. Analytical resolution

1. Using the magnitude and direction of force \mathbf{A} you entered in Table 3.1, calculate the components A_x and A_y . Record these values in Data Table 3.3.
2. Repeat step 1 to resolve analytically force \mathbf{B} .
3. Repeat step 1 to resolve analytically force \mathbf{C} .

A3. Experimental resolution

Set up the force table on its three legs. Adjust the legs so that the table is perfectly horizontal (use a level in orthogonal directions to be sure).

Vector \mathbf{A} :

1. Clamp pulleys at the 40° mark, at the 180° mark, and at the 270° mark on the rim of the force table. Adjust each pulley wheel so that its top is slightly above the force table top, then tighten the pulley clamp to keep the pulley at that height. **Do not over-tighten the pulley clamp!**
2. Tie three strings securely to the white plastic ring, place the ring around the pin at the center of the table, run the strings over the three pulleys, tie loops in their lower ends, and hook mass hangers in the loops.
3. Place a total of 50 g (including the hanger!) on the string at 40° . This string will apply force \mathbf{A} to the ring. The string at 180° will apply the *equilibrant* force to component A_x and the string at 270° will do the same for component A_y . Readjust the pulley wheel clamps, if necessary, to ensure that all three strings are parallel to the force table top, and make sure that they fit in the grooves of the pulley wheels.
4. Write A , A_x , and A_y in the first three rows of the *Vector* column of Table 3.4. The mass of each hanger is stamped on it. Convert each mass value to kg and record these masses in the second column of Table 3.4.
5. Place additional masses on the A_x and A_y hangers until the ring appears to be in equilibrium. When it does, remove the pin, displace the ring a bit, and tap it. The ring will return to its central position when you have achieved true equilibrium. Determine the additional mass you placed on each mass hanger. Convert each of these masses to kg and record their values in Table 3.4.
6. Calculate the experimental values of forces A_x and A_y (in N) and record these values in Table 3.4. (Recall that the equilibrant forces applied by the strings are just the weights of the total masses suspended from the 180° and 270° pulleys, respectively, and that the components A_x and A_y have the same magnitudes as their equilibrants.)

- Using the analytical values in Part A2 as the true values, calculate the errors in the experimental values of A_x and A_y . Record the average error value in Table 3.4.

Vector B:

- Starting with a clean force table, clamp pulleys at the 0° mark, the 120° mark, and the 270° mark and run the three strings over them. Place a total of 70 g (including the hanger!) on the string at 120° . This string will apply force **B** to the ring.
- The string over the 0° pulley represents the equilibrant force to B_x , and the string at 270° represents the equilibrant to B_y . Hook a mass hanger to each of these strings. Write B , B_x , and B_y in the next three rows of the *Vector* column of Table 3.4.
- Place masses on the 0° and 270° strings until the ring appears to be in equilibrium. When it does, remove the pin, displace the ring a little, and tap it. The ring will return to its central position when you have achieved true equilibrium. Determine the additional mass you placed on each hanger. Convert each of these values to kg and record them in Table 3.4.
- Calculate the experimental values of the components B_x and B_y (in N) and record these values in Table 3.4.
- Using the analytical values from part A2 as the true values, calculate the errors in the experimental values of B_x and B_y and record the average value in Table 3.4.

Vector C:

13. Repeat vector **B** steps 1–5 with 100 g (including the hanger!) for force **C** with the pulley at the 230° mark, and the C_x and C_y pulleys at 0° and 90° .

B. Adding Vectors

B1. Graphical addition

- Using graph paper and a straightedge, draw x and y axes. Label the origin as point O and label both axes. Choose a scale (such as 1 in = 2 N) with which you will use to draw the vectors.
- Using your straightedge, dividers, and protractor, draw the arrows representing forces **A**, **B**, and **C** (the same forces used in part A). Start at the origin so that the three arrows are head-to-tail.
- Using a straightedge, draw the resultant **R** from O to the tip of **C**. Find the magnitude of **R** by measuring its length and comparing it with your scale. Find the direction of **R** by measuring with a protractor the angle it makes with the positive x axis. Record the values of the magnitude and direction of **R** that you have obtained graphically in Table 3.5.
- Using the analytical values you find in section B2 as true values, calculate the percent error in the graphical value of the magnitude of **R** and calculate the *absolute difference* between the graphical value and the true value of the direction of **R**. Record these error values in Table 3.5.

B2. Analytical addition

- Write **A**, **B**, **C**, and **R** in the four rows of the first column of Table 3.6. Calculate the x and y components of each of the four forces (in N). Record each value in the second and third columns of Data Table 3.6.

2. Calculate the magnitude of the resultant **R**; record its value in the fourth row, fourth column of Table 3.6.
3. Calculate the angle θ that the resultant **R** makes with the x axis and record its value in Table 3.6. Use this resultant (magnitude and direction) for the true value when you calculate errors in **R**.

B3. Experimental addition

1. Starting with a clean force table, clamp three pulleys at the angular positions corresponding to the directions of forces **A**, **B**, and **C**. Run the three strings from the ring over the pulleys and hook a mass hanger on each string. Place the appropriate mass on each of the hangers to create the forces **A**, **B**, and **C** on the ring. Record the hanging mass and the magnitudes and directions for the three given vectors in Data Table 3.7.
2. Tie a fourth string to the ring. By positioning this string and pulling on it, determine the direction and force required to keep the ring in equilibrium. This string represents the equilibrant force **E**.
3. Place a fourth pulley where the **E** string crosses the edge of the table but don't clamp it yet. Run the string over this pulley and hook a mass hanger on it. Add masses and fine tune the pulley position until the ring appears to be in equilibrium. To test it, remove the pin, displace the ring a little, and tap it. The ring will return to its central position when you have achieved true equilibrium. Record the hanging mass and the magnitude and direction of **E** in the fourth row of Table 3.7.
4. Calculate and record the magnitude and direction of **R** in the fifth row of Table 3.7. Using the analytical values you calculated in Part B2 as the true values, calculate the percent error in the magnitude of **R** and the absolute difference in the direction of **R**. Record these values in the fifth and sixth columns of Table 3.7.
5. Disassemble all the equipment, dispose of the string, return all equipment to the setup cart, and clean up around your lab table area.