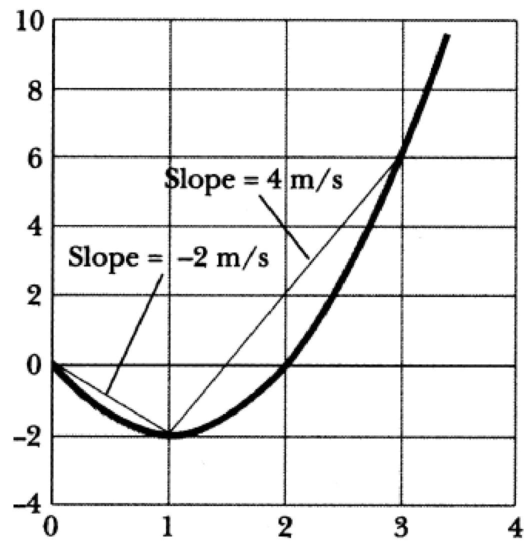
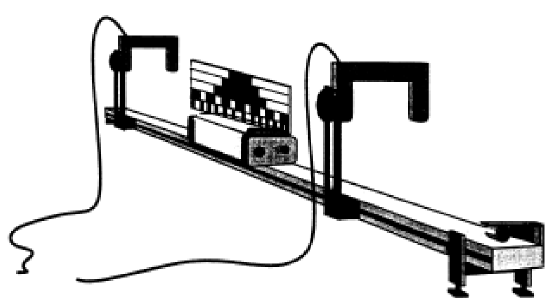
Velocity & Acceleration





Produced by the Physics Staff at
Collin County Community College

Purpose

You will investigate the motion of objects moving with constant acceleration.

Equipment

- 1 Pasco track
- 2 Photogates w/ brackets
- 2 Motion sensor
- 1 Rod clamp
- 1 Large ringstand
- 1 Meter stick
- 1 Leveling foot
- 1 End stop
- 1 Plunger cart
- 1 Small picket fence

Introduction

At first glance, it seems obvious from our everyday experience that here on Earth (1) inanimate objects at rest do not move of their own accord, and (2) if you put them into motion they soon return to rest. Based on such observations, Aristotle developed a theory that any object's natural state is to be at rest and that being in motion is an unnatural (temporary) state. His theory was accepted by the world's philosophers for over two thousand years.

But Galileo and Newton asked questions about the motion of objects that could not be answered by Aristotle's theory. In seeking answers to their questions, they ultimately produced a new theory of motion that applied not only to all objects here on Earth, but to everything in the heavens as well. This theory has since been developed more fully and is now accepted by educated people all over the world.

Our current view of motion is that, when all interactions between an object and its environment are considered, any object in constant-velocity motion is just as much in a natural state as is an object at rest. For an isolated object (an object that does not interact with its environment through friction or air drag), no effort or force is required to maintain its constant motion, just as no effort is required to keep it at rest. We can only imagine such an isolated object, of course, because in real life there is always some small friction or drag acting on moving objects on Earth.

The tendency of any object, then, is to remain in whatever state of constant motion it is already in (including constant zero motion – that is, being at rest) as long as no external force is acting on it. This tendency to avoid any change in motion is due to the object's inertia. Because of an object's inertia, a net force must be exerted on it to make its state of motion change – from rest to moving, from moving to rest, from moving at one speed to moving at another, or from moving in one direction to moving in another.

The relationships between an object's position, its speed, its acceleration (rate of change of speed), and time are investigated in a branch of physics called kinematics. The simplest case, the one you will examine in this experiment, is motion in a straight line – called one-dimension motion.

In this experiment, you will

1. Become familiar with some basic measurements of motion.
2. Learn to make and read graphs of position, speed, and acceleration vs. time.
3. Learn the distinction between average and instantaneous speed and acceleration.
4. Investigate one-dimensional motion at constant acceleration.

Theory

Dynamics of Motion

For an object in motion, its position is a function of time [$x = f(t)$]. You can completely describe the motion of any object at any time (in the past, the present, or the future) by describing this function; i.e., by stating the quantitative relationship between its position, speed, and time.

Although a moving object's position varies with time, its speed and acceleration can be either variable (can vary with time) or constant. Furthermore, since any object's position is relative to some coordinate system, its motion is also relative – what appears as motion to one observer may appear as rest to another observer if the two observers are moving with respect to one another.

Speed (Velocity)

If an object's position varies with time, we say it is in motion. If the position is constant in time, the object is at rest. We define an object's displacement as the quantity of change of its position, and its speed as the rate of change of its position. We write the relationship between speed (v), position (x), and time (t) symbolically as

$$\bar{v} = \frac{\Delta x}{\Delta t}$$ ***Equation 2.1***

where $\bar{v}$ is the object's *average* velocity, Δx is its displacement and Δt is the time interval during which the position changed. The measurement unit for speed is simply the ratio of distance units to time units. It can be miles/hr, meters/min, feet/sec, furlongs/fortnight, or whatever.

But this simple relationship doesn't give us all the details of the object's motion. The most obvious question left unanswered is, "Did the object's velocity vary during the time interval Δt, or did it remain constant?" If the velocity remained constant, then you know its value was $\bar{v}$ at every instant of time during the interval. But how do you find the value of velocity at any instant if it varied during the time interval? This question leads us to two concepts of velocity.

The first concept is called average velocity. It is the ratio of the object's displacement to the time interval during which it moved. Equation 2.1 expresses average velocity. For example, it's easy to determine how long a 200-mile trip will take if you know that your average velocity on it will be 50 miles per hour.

On the other hand, the highway police don't care what your average velocity is over the 200 mile trip. They want to know how fast you are driving at the instant they are observing you (when their radar beam strikes your car), so they can decide whether or not you are exceeding the speed limit at that particular point in time. The highway cop wants to know your *instantaneous* velocity.

Your car's speedometer indicates your instantaneous *speed* – your speed at every instant of time. Your trip odometer integrates this instantaneous speed over the time interval of the trip to calculate your total trip distance. Equation 2.2 below shows what happens if you calculate the average velocity of a moving object over smaller and smaller time intervals. The value of the object's average velocity approaches the value of its instantaneous velocity – the derivative of displacement with respect to time.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$ ***Equation 2.2***

Acceleration

Another question unanswered by these two equations for velocity is, "When the velocity is changing, how fast is it changing?" In other words, what is the rate of change of the velocity? Just as we call the rate of change of position velocity, we call the rate of change of velocity *acceleration*.

The measurement unit for acceleration is the ratio of velocity units to time units: miles/hour/sec, meters/min/sec and so forth. However, it is customary in this ratio to use the same unit for time as we use in the velocity ratio, leading to the square of that unit in the denominator (meters/sec^2, etc.).

Galileo determined that, in the absence of air drag, all freely-falling objects fall with the same constant acceleration, i.e., their velocity increases steadily until they hit the ground, the floor, or whatever. This constant acceleration near the earth's surface is given is own symbol, *g*, and its value is $g = 9.80$ m/s^2.

Because he didn't have the accurate interval timers we have today, Galileo cleverly used a timer that was always available, albeit not very stable – his heartbeat. Even so, he had to devise a method of slowing the descent of the object so its fall lasted more than one or two heartbeats. He realized that an object sliding down an (imaginary) frictionless inclined plane (a ramp) would also move with a constant acceleration, as would a ball rolling down the ramp. He also saw that the acceleration of such an object would be a fixed fraction of its free-fall acceleration, *g*, depending on the inclination angle of the ramp.

The distance fallen by a freely-falling object, Δy, is related to its initial speed v_0, the time of fall Δt, and of course its acceleration due to gravity *g*. This relationship is:

$$\Delta y = v_0 t - \frac{1}{2} g t^2$$

where we assume that $t_0 = 0$. If the object starts its fall from rest (i.e., if $v_0 = 0$), the first term on the right drops out and the distance it falls is simply:

$$\Delta y = -\frac{1}{2} g t^2$$

If a moving object's acceleration varies (not a freely falling object, of course), you can write average and instantaneous values of its acceleration just as you wrote them for velocity. The average value of acceleration over the time interval Δt is:

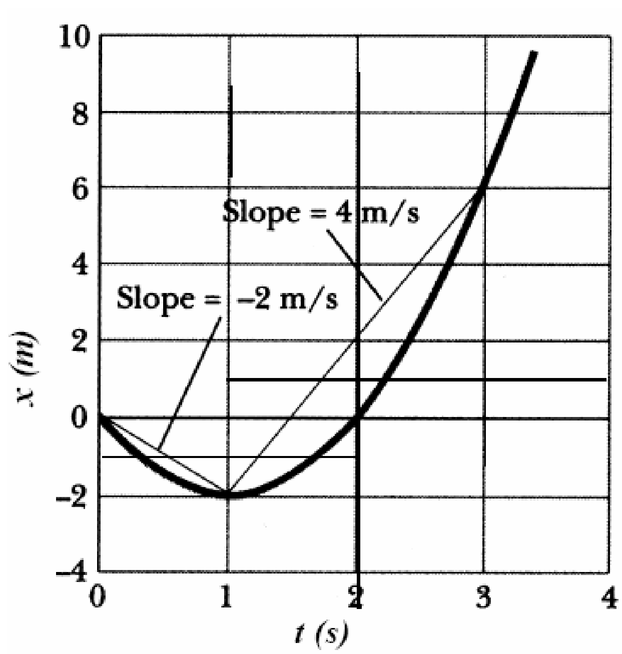
$$\overline{a} = \frac{\Delta v}{\Delta t}$$ ***Equation 2.3***

In the limit as $\Delta t \to 0$, the average acceleration approaches the instantaneous acceleration at the time t, it is simply the derivative of velocity with respect to time at that instant in time.

$$a(t) = \frac{dv}{dt}$$ ***Equation 2.4***

<u>Graphing the Motion</u>

The relationships between position *x*, time *t*, velocity *v*, and acceleration *a* can be described analytically, as in equations 2.1 – 2.4, or graphically, as in the figure below. A graph of an object's position vs. time allows you to easily determine its average and instantaneous velocity. You can write the equation defining *v* as $\Delta x = vt$. If $x_0 = 0$, the equation becomes $x = vt$. A graph of *x* vs. *t*, such as shown, is a gold mine of information about the object's motion.



From this graph, we can see that the object's position when $t = 0$ is $x = 0$. Then it starts moving in the minus *x* direction and reaches $x = -2$m when $t = 1$s. After that, it reverses direction and is back at $x = 0$ when $t = 2$s. It continues moving in the positive *x* direction, reaching the position $x = 6$m when $t = 3$s.

By drawing chords between specific points on the curve, we can find the object's average velocity over a given time interval. In the example shown, the slope of the chord between any two points on the graph is equal to the average value of the velocity during that interval. For $0 < t < 1$s, the average velocity is – 2m/s, and for $1\text{s} < t < 3$s, the average velocity is + 4m/s.

As the time interval shrinks to a single point on the *t* axis, the chord becomes the tangent to the graph (the average velocity becomes the instantaneous velocity) at that instant in time.

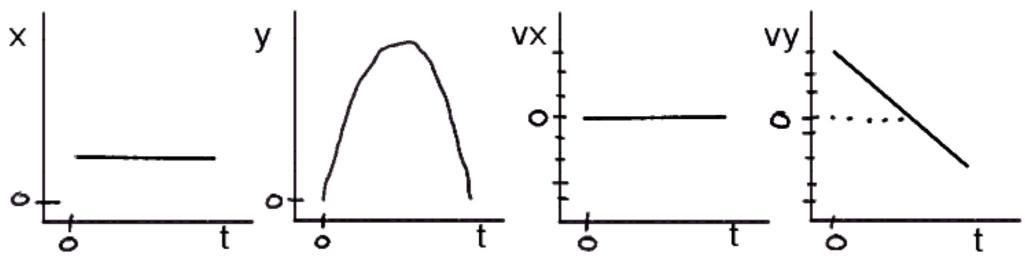
If the velocity were constant, the slope of the graph would be constant (the graph would be straight). A positive slope indicates a positive value of *v* (the object's displacement, *x*, is increasing with time), a negative slope indicates a negative value of *v*, and a changing slope indicates a changing value of *v*.

From differential calculus, we see that the value of the slope at a specific time (the instantaneous velocity at that time) is simply the derivative of the position with respect to time.

Similarly, a graph of Δv vs. Δt will be a curve whose slope at any point is equal to the acceleration *a*. If *a* is constant, the slope of the graph will be constant (the graph will be straight). A positive slope indicates a positive value of *a* (the velocity is increasing with time), a negative slope indicates a negative value of *a* (the velocity is decreasing with time), and a changing slope indicates a changing value of *a*.

The value of the slope of the *v* vs. *t* graph at a specific time (the instantaneous acceleration at that time) is the derivative of the speed with respect to time.

Even if motion is considered in two dimensions, graphs of position vs. time and velocity vs. time can be drawn for dimension individually. For example, the graphs below are for an object thrown straight up then falling back down again.



Procedure

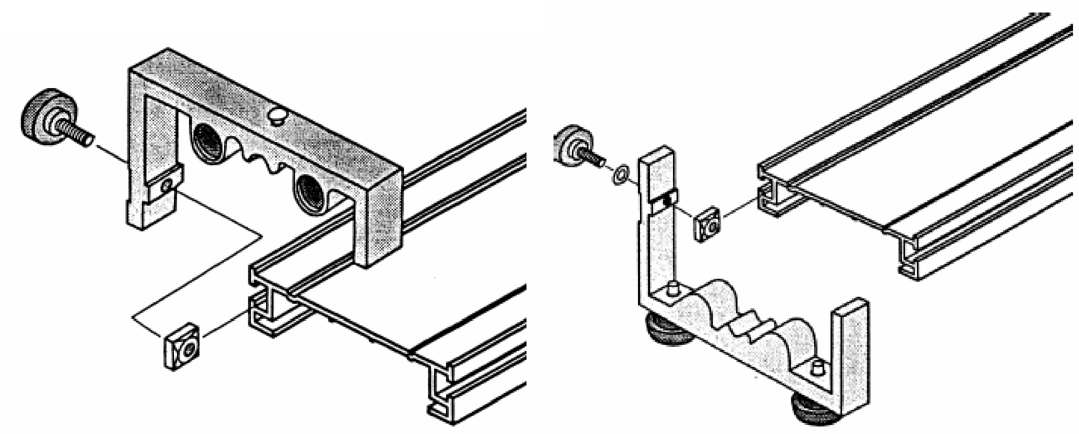
With the power to the computer system switched off, connect the following sensors to the Interface:

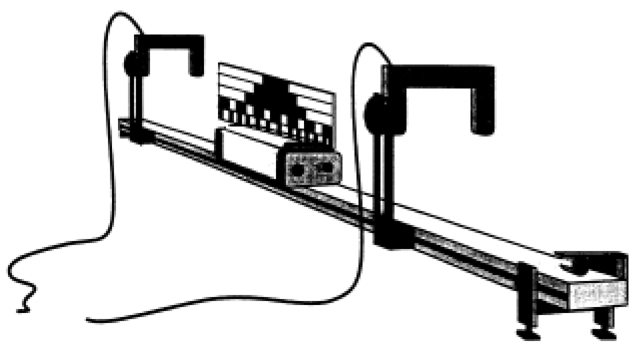
1. Plug one photogate sensor into Digital Channel 1.
2. Plug another photogate sensor into Digital Channel 2.
3. Plug the motion sensor plugs into Digital Channels 3 and 4 (the yellow plug into Channel 3).

A. Average and Instantaneous Speed

You will use two photogate sensors a known distance apart to measure the time it takes for an assumed frictionless cart to roll down a slightly inclined track from one photogate sensor to the other. Knowing the distance and time, the computer will calculate the cart's average speed during the time interval it is between the sensors.

You will then reduce the distance between photogates several times and repeat the process for each distance. You will then plot the cart's average speed vs. the distance between sensors, and you will use this plot to estimate its instantaneous speed at the midpoint between the sensors.



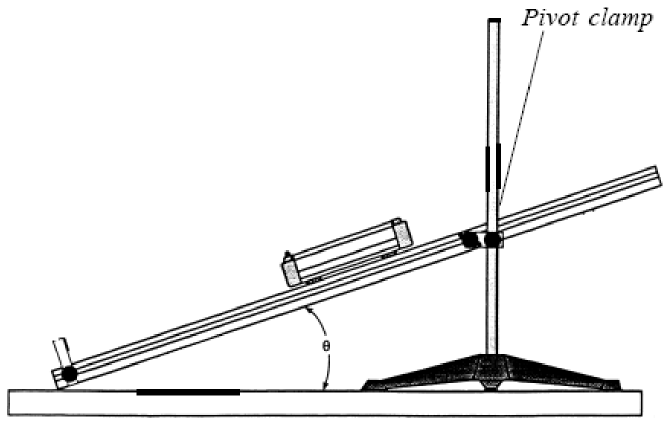


1. Place the track on the lab table and insert a leveling foot at one end of the track by aligning the square nut within the groove on either side as shown above. To minimize sag in the track, slide the leveling foot in by about 50 cm from the end of the track. The track should now be slightly inclined.
2. Install an adjustable end stop on the bottom end of the track as shown above.
3. Using a soft lead pencil, print an x_1 near the midpoint of the track and record its position as read on the track's scale under Table 2.1. Print an x_0 near the top end of the track to be the starting line. The cart must be released from this same spot in each data run.
4. Using the brackets, mount the channel 1 photosensor at a position 40.0 cm toward bottom end of the track from x_1 and the channel 2 photosensor at a position 40.0 cm toward the top end of the track from x_1 as shown above. Measure both distances to the nearest 0.1 cm.

5. Mount the small plastic "picket fence" in the slot on the cart (with the solid black rectangle on top) and place the cart on the track. Adjust the height of the two photogates so that the top black rectangle on the picket fence interrupts the photogate beam as the cart passes through. Remove the cart from the track.
6. Switch on the Computer and Interface, and open Data Studio. In the Data Studio window, select Create an Experiment.
7. From the list of sensors, double-click the Photogate icon. This configures Channel 1 for the first photogate. To configure Channel 2 for the second photogate, double-click the Photogate icon again.
8. Click the Timers button in the Experiment Setup window. In the Timer Setup window under Timing Sequence Choices, click the down arrow for Channel 1 and select Blocked. This tells Data Studio to start timing when the photogate plugged into Digital Channel 1 is blocked by the picket fence.
9. Now click the down arrow for Channel 2 and select Blocked again. This tells Data Studio to stop timing when the photogate plugged into Digital Channel 2 is blocked. Type *Time Between Gates* as the label. Click the Done button to close the Timer Setup window.
10. In the Displays list, double-click on Digits, select *Time Between Gates*, and drag the digits display to the right.
11. Do a practice run to familiarize yourself with the setup. Hold the cart at the top of the track with its front at x_0. Click the Start button, and release the cart cleanly so it rolls down the track with zero initial velocity. Data recording begins when the picket fence blocks photogate 1 and ends when it blocks photogate 2.
12. The cart's time between gates will appear in the digits display and run #1 will appear in the Data List. You can erase it by selecting Run #1 and pressing Delete.
13. For your first actual data run, repeat steps 11-12. After the cart stops, remove it from the track. Record the time between gates (from the digits display) and calculated average speed (distance between gates/time between gates) for the 0.80 m distance in Table 2.1.
14. Move each photogate exactly 5.0 cm inward toward x_1 and repeat steps 11-13 for a 0.70 m distance. Continue to decrease the distance between photogates by equal amounts in 10-cm steps, repeating steps 11-13, and recording each time and speed in Table 2.1.
15. Plot a graph of average speed vs. distance between photogates and extrapolate the curve to a distance of zero. The y intercept is the cart's instantaneous speed at the point x_1. Record this speed in Table 2.1
16. Calculate the percent differences between the instantaneous speed at x_1 and the average speed in each trial. Record these percent differences in Table 2.1.
17. Remove the track foot and photogates and set them aside. Leave the adjustable end stop in place.

B. Motion at Constant Acceleration

In this part, you will use the motion sensor to determine the time variation of the cart's position, velocity, and acceleration as it rolls down the inclined track. Data Studio will calculate the cart's velocity and acceleration by taking the first and second derivatives of its changing position. You can use a protractor to find the track's inclination angle.



1. Fix the pivot clamp on the track about 50-cm from the top end of the track. Insert the ring stand into the pivot clamp and raise the top end of the track, as shown above, so that its top end is about 40.0 cm above the table. The other end of the track must rest directly and squarely on the table.
2. Measure the inclination angle θ, then record θ and sin θ in Table 2.2.
3. Place the motion sensor on the table just beyond the stop on the lower end of the track and tilt it to aim it up the track.
4. With the Data Studio window active, click *File* then *New Activity* from the menu bar. Select *Create Experiment*. Don't save any files. Next, create a velocity graph (velocity vs. time).
5. Run a practice trial to familiarize yourself with the setup. Hold the cart at x_0 with the plunger facing down the track, click the start button in Data Studio, then release the cart cleanly (with zero initial velocity). Data recording and graphing will continue *until you press stop*. Run #1 will appear in the Data list. You can erase this practice run by highlighting Run #1 then pressing Delete.
6. Repeat step 5 for your first run of actual data recording. Click anywhere in the velocity graph to select it, highlight the linear portion of the graph, then click the *Fit* button on the toolbar and select *Linear Fit* from the menu. The graph itself shows the relationship between v and t in graphical form. The equation $y = a1 + a2x$ derived from the curve fit shows the same relationship in analytical form. The *a1* term in the equation is the value of v when $t = 0$, and the *a2* term is the value of the slope of the graph. Since the selected portion of the graph is a straight line, the value of *a2* (the slope of the curve) is a constant, and is equal to the acceleration a.
7. Record the (constant) value of a in Table 2.2.
8. Lower the top end of the track to about 35.0 cm, and repeat steps 2, 5, 6 and 7 for your second run of actual data recording.
9. Continue repeating this procedure, lowering the zero end of the track in five-centimeter increments until the height is about 5.0 cm. Record all the values of θ, sin θ and a in Table 2.2.
10. Plot a graph of a vs. sin θ. Find and record the value of a at sin θ = 1.0.
11. Quit the Data Studio program. **Don't save any files.** Shut down the computer, disassemble the equipment, and place it *neatly* back on the equipment cart.