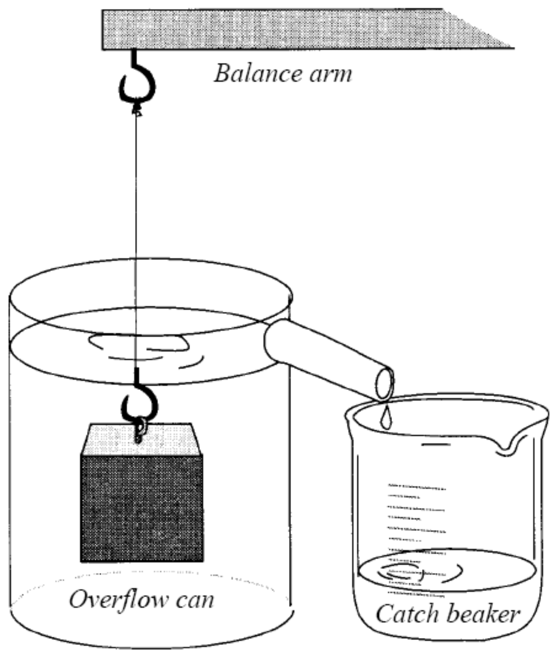
Archimedes' Principle



Produced by the Physics Staff at
Collin County Community College

Purpose

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids.

Equipment

- 1 Force Sensor
- 1 Overflow Can
- 1 Large Ringstand
- 1 Metal Cube w/ Hook
- 1 Wooden Object w/ Hook
- 1 Metal cylinder w/ hook
- 1 Lab Jack
- 1 500-mL, 1 1000-mL Beaker
- 1 Support Rod w/ 90° Clamp
- 1 Lab Balance & Roll of String
- 1 Hydrometer
- Bottle of Unknown Liquid

Introduction

A solid object may float or sink when placed in a given fluid (i.e., in a gas or a liquid). If the object floats, it is buoyed up by a force equal to its weight. According to the ancient Greek philosopher Archimedes, the buoyant (upward) force exerted on an object that is either wholly or partially submerged in a fluid is equal to the weight of the amount of fluid displaced by the object. The object will sink when its weight exceeds the weight of the displaced fluid.

Because of Earth's gravity, every fluid substance from the atmosphere to the oceans to a tank full of gasoline has an internal pressure that increases with depth. And because fluids are compressible, the greater pressure at a given depth squeezes the fluid into greater density at that depth.

We live our lives at the bottom of the atmospheric ocean which exerts about 15 pounds of force on every square inch of our bodies. We don't notice this pressure, of course, because the fluids inside our bodies exert a balancing outward pressure. This balance is not automatic. Our internal pressure doesn't adjust to external pressure changes, so we must surround our bodies with air at normal atmospheric pressure if we ascend into the atmosphere or descend into the ocean.

Because of the vertical pressure gradient in fluids, the pressure on the lower surface of a submerged object is always greater than that on the upper surface. This is the root cause of the buoyant force.

The term *specific gravity* is probably more confusing than it is worth. Specific gravity is really nothing but relative density; i.e., it is a unit-less ratio between the density of an object or a substance relative to the density of pure water, with both values of density expressed in the same units.

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids. When you finish the experiment, you will be able to

1. Determine whether or not an object will float or sink in a fluid if you know the density of each.

2. Explain the difference between density and specific gravity.
3. Determine the density or specific gravity of a solid or fluid whether it floats or sinks.

Theory

The buoyant force is described by Archimedes' principle as: *an object, when placed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.* The principle applies to an object either entirely or partially submerged in the fluid. The magnitude of the buoyant force depends *only* on the weight of the displaced fluid, and not on the object's weight. Using Archimedes' principle, you can deduce that an object:

1. will float in a fluid if the object's density is less than the fluid's density ($\rho_o < \rho_f$).
2. will sink if the object's density is greater than the fluid's density ($\rho_o > \rho_f$).
3. will remain in equilibrium at a given submerged depth if the object's density is exactly equal to the fluid's density at that depth ($\rho_o = \rho_f$).

The buoyant force on a floating object F_b is related to the properties of the displaced fluid by:

$$F_b = m_f g = \rho_f V_o g$$

where ρ_f is the density of the fluid, V_o is the volume of the submerged part of the object, and g is the acceleration due to gravity.

The volume of the submerged part of a cylinder oriented vertically is equal to its cross-sectional area A multiplied by the height h of the submerged part, so the buoyant force on it is:

$$F_b = m_f g = \rho_f Agh$$

This is a linear relationship between F_b and h, so if you lower the cylinder into a fluid as you measure its weight, then plot F_b vs. h, the slope of the plotted straight line will be $\rho_f Ag$, i.e., directly proportional to the density of the fluid. This is a cool way to determine the density of an unknown fluid.

You can determine the density of an unknown solid object in a similar fashion. It's easy to measure the mass of an object, but unless it has a regular shape it's not so easy to measure its volume. But Archimedes showed us how to measure volume by measuring weight.

When the object is completely submerged in water, its weight (but not its mass) will decrease by an amount equal to the upward buoyant force the water exerts on it. So:

$$\Delta W_o = W_o \text{ (in air)} - W_o \text{ (in water)}$$

This upward force is also equal to the weight of the displaced water, or:

$$\Delta W_o = W_w = m_w g = \rho_w g V_w$$

But the volume of the water is equal to the volume of the object, so:

$$V_w = V_o = \frac{\Delta W_o}{\rho_w g}$$

The density of the object is therefore:

$$\rho_o = \frac{m_o}{V_o} = \frac{m_o \rho_w g}{\Delta W_o}$$

You can also determine the density of an unknown liquid without measuring the submerged height of the solid object. With an object with density greater than that of the unknown liquid, first weigh it in air, then when it is submerged in the liquid, and then when it is submerged in water. By an analysis identical to that for the density of a solid object, you can show that:

$$\rho_o = \frac{\Delta W_o \text{ (in liquid)}}{\Delta W_o \text{ (in water)}} \rho_w$$

Procedure

You will use a lab balance to weigh a metal object in air and under water, to weigh a wooden object in air and under water, and to weigh the metal object under water and under an unknown liquid. From these measurements, you will calculate the density of the metal object, of the wooden object, and of the unknown liquid. In Part D, you will use the computer force sensor to measure the weight loss of a metal cylinder as you slowly submerge it in water. You will plot F_b vs. h and determine the density of water.

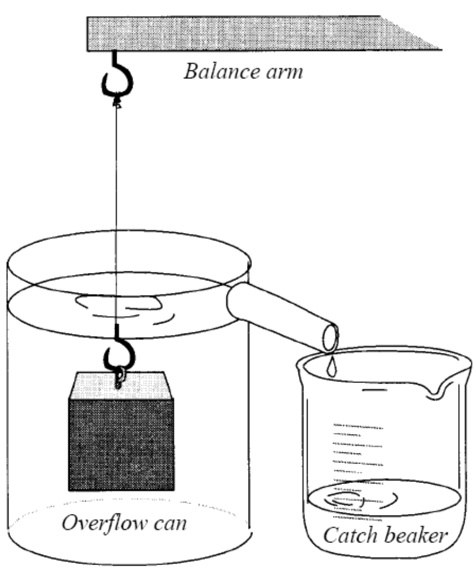


Figure 12.1. Determining volume of object

A. Density of Metal Object

1. Mount the lab balance on a ringstand about two feet high and place the overflow can under the balance pan on a lab jack.
2. Weigh the metal object m_m and the dry catch beaker m_b, and record their masses in Data Table 12.1.
3. Place the catch beaker beneath the spout of the overflow can as shown in Figure 12.1, and fill the overflow can with water until it runs out the spout. Then empty and replace the beaker.
4. Hang the metal object by a short thread from the hook beneath the balance pan, then raise the lab jack until the object is completely sub-merged in the water. Weigh the submerged object m_{ms} and record its mass in Table 12.1.
5. Remove the sample from the hook under the balance and measure and record the combined mass of the catch beaker and the overflow water m_{comb}.
6. Record the measured mass m_{H2O} and the weight W_{H2O} of the displaced water.
7. From Archimedes' principle, calculate and record the weight of the displaced water.
8. Calculate and record the percent difference between the measured and calculated weights of the displaced water.
9. Calculate and record the density ρ_m of the metal object. Use the accepted value of the metal density (consult your textbook for this value) to calculate the percent error.

B. Density of Wooden Object

1. Weigh the wooden object and record its mass m_{wood} in Table 12.2. Then place the balance on the ringstand.
2. Attach a metal sinker m_{sink} to the bottom of the wooden object (the sinker should weigh enough to submerge the object completely) and hang the combination from the balance hook.
3. Place enough water in the large beaker so that the wooden object and sinker can be submerged completely without the sinker touching bottom. Place the beaker on the lab jack.
4. Raise the lab jack to submerge *only the sinker*, then measure and record the mass m_1 of the object/sinker combination.
5. Raise the lab jack to submerge *the object/sinker combination*, then measure and record their combined mass m_2.
6. Calculate and record the wooden object's submerged mass $m_{wood\text{-}s}$ in water.
7. Calculate and record the buoyant force F_b on the wooden object.
8. Calculate and record the density ρ_{wood} of the wooden object.

C. Density of Unknown Liquid

1. Hang the metal object m_m from the balance hook.
2. Measure and record in Table 12.3 the object's mass m_{ml} when submerged in the unknown liquid, then when it is submerged in water m_{mw}.
3. Calculate and record the density ρ_l of the unknown liquid.
4. Pour the unknown liquid into a graduated cylinder and measure its density with the hydrometer. Record it in Table 12.3.
5. Calculate and record the percent difference between the calculated and measured densities of the liquid.

D. Density of Water

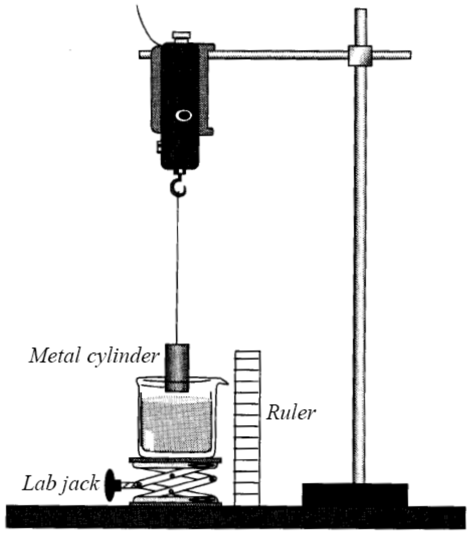


Figure 12.2. Measuring F_b vs. h

1. With the power switched off, connect the Force Sensor cable to Analog Channel A.
2. Switch on the computer system, open Data Studio, and create new experiment. Drag the Force Sensor icon to Analog Channel A. Open a Digits Display, and select Force (Channel A).
3. Mount the force sensor on the ringstand as shown in Figure 12.2. Measure the mass m_c of the cylinder using the mass balance. Hang the aluminum cylinder from the force sensor with a string.
4. Place enough water in the large beaker so that the cylinder can be submerged completely without touching bottom, and place the beaker on the lab jack below the hanging cylinder (see Figure 12.2). The bottom of the cylinder should be just touching the water.
5. Position a vertical meter stick next to the lab jack and record the initial position of the jack top. Before recording data for later analysis, you may wish to practice using keyboard sampling to collect data.
6. Press the tare button on the side to zero the sensor.
7. Click the Start button. The Force F_c (the cylinder's apparent weight) will appear in the Digits display. Record this value of the force in Table 12.4.
8. Immerse the cylinder 5 mm by raising the beaker of water 5 mm with the lab jack. Record the new value of force from the Digits display at a depth h of 0.005 m.
9. Increase the submersion depth in 5 mm steps. After each step increase, wait for the force reading to stabilize then record the force value from the Digits display.
10. Repeat the data recording procedure until the top of the cylinder is submerged, then stop data recording by clicking the Stop button.
11. Plot a graph of F_b vs. h. Record the value of the slope in Table 12.4. Calculate and record the density of the water.
12. Compare your experimental value of water's density to the accepted value and record the percent error.
13. Close the computer, disconnect the sensor, coil the sensor cable, and secure it with a twisty-tie. Return your equipment to the lab cart and clean your area of the lab table.