UNIVERSITY PHYSICS

EXPERIMENT 11

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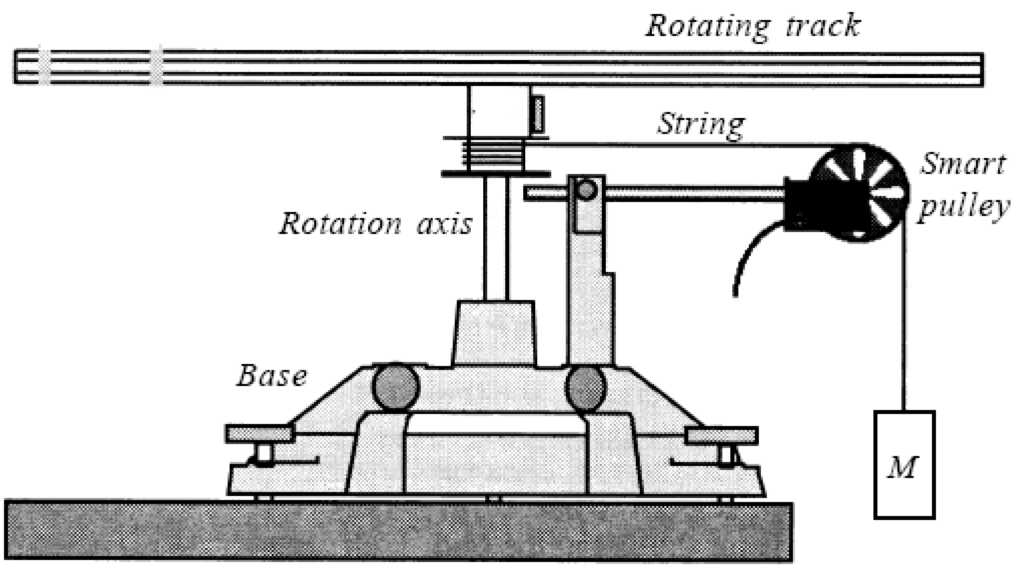
COLLIN COUNTY COMMUNITY COLLEGE

Conservation of Angular Momentum



Produced by the Physics Staff at
Collin County Community College

Purpose

The Law of Conservation of Angular Momentum is: *If a system has angular momentum L about a specified axis, the magnitude and direction of L will remain fixed unless a torque about that axis is applied from outside the system*. In this experiment, you will use this conservation law to solve for initial conditions based on final conditions that you measure.

Equipment

- 1 Rotating Platform w/ Support Rod
- 1 Projectile Launcher w/ Steel Ball
- 2 Photogates w/ Bracket
- 1 Vernier Caliper
- 1 Smart Pulley
- 1 Projectile Catcher Accessory
- 1 Slotted Mass Set w/ Hanger
- 1 Lab Balance
- 1 C Clamp
- Rubber Bands and String

Introduction

So far in your study of motion you have learned that you can describe angular motion using quantities that are directly analogous to the quantities used for linear motion – angular displacement analogous to linear displacement, angular velocity analogous to linear velocity, etc.

Both types of motion are defined with respect to some coordinate system. In linear motion, an object's instantaneous position, velocity, acceleration, and the net force acting on it are all defined in terms of either a Cartesian or a polar coordinate system. But even though the values of these quantities depend on the coordinate system, they do not depend on an object's radius vector **r** (the vector distance from the origin to the object). For example, an object's linear velocity does not depend on its radial position **r**.

In angular motion, however, all motional properties of a rigid object (but not of a fluid) depend on **r**, the distance and direction of the object's center of mass from the axis of rotation. If the angular motion is circular, the axis of rotation is fixed at the origin. If the motion is not circular, the position of the axis varies from point to point anywhere in space except on the path of motion, and the direction of the axis is always normal to the plane of the path of motion.

In general, there are two kinds of angular motion, depending on where the axis is. The motion is called rotation when the axis is located within the object, and it is called revolution when the axis is outside the object.

The angular analog to linear momentum **p** is called, not surprisingly, angular momentum **L**. An object does not have to be in angular motion to have angular momentum. In fact, an object moving in a straight line has a varying angular momentum about any point not on the line, as shown in Figure 11.1.

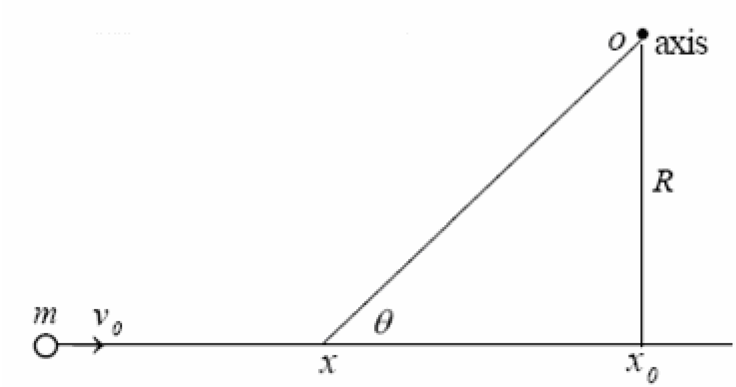


Figure 11.1. Top view of ball moving past axis

In this experiment you will study the rotational inertia and the angular momentum of objects in both linear motion and rotational motion. After you complete this experiment you should have a working understanding of rotary inertia, angular velocity, and angular momentum.

Theory

Angular Momentum In Linear Motion

Imagine a ball with mass m launched horizontally in a straight line with velocity v_o as shown in top view in Figure 11.1. At every point x along its path, the ball has an angular momentum L about an axis normal to the page located at point O a normal distance R from its path.

$L = mv_oR \sin \theta$

L has its greatest value, of course, at x_o, its point of closest approach to the axis, where θ = 90°.

Imagine that this ball strikes the end of a rotary track lying along R. The track is initially stationary but is free to rotate in a horizontal plane about the axis O. Finally, imagine that the ball sticks to the track, causing the track and ball to rotate about the axis at a measurable angular speed ω.

Knowing these properties and relationships, can you determine the initial velocity of the ball? Your answer should be yes. Let's step through the analysis that leads you to the value of the initial velocity.

This is a collision problem, and in any collision the total momentum (both linear and angular) of the system (the vector sum of the momentum of all the system's components) is conserved. The system you are considering, of course, consists of two components — the moving ball and the (initially stationary) track. Since the collision is completely inelastic, the system's mechanical energy is not conserved during the collision. However, its angular momentum is conserved — its initial value is equal to its final value.

Because the track is initially stationary, the initial angular momentum of the system about the axis is simply the initial angular momentum of the ball about the axis, as given in the equation above. After the collision, the total angular momentum of the system about the axis is defined as

$$L = I\omega$$

where I is the rotary inertia of the combined ball-track system about the axis, and ω is the angular velocity of the system after the collision.

With these two relationships, you need only eliminate L between the equations to solve for the initial velocity of the ball:

$$v_o = \frac{I\omega}{mR}$$

So to calculate v_o, you need to know the values of the four quantities on the right side. You can easily measure m and R, but how do you find the values of ω and I?

Well, you can measure ω (in rad/s) directly with a Rotary Motion sensor or, if this sensor is not available, you can calculate it from the rotational period:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where f is the rotational frequency (in rev/s), and the period T is easily measurable with a stop watch.

To find the value of I, start by writing Newton's second law for angular motion as: $\tau = I\alpha$.

So all you need to do is apply a known torque τ to the track and measure the resulting angular acceleration α. You can create the torque by winding a string around the track's axle of radius r and pulling on it with a known tension force F_T. The applied torque is then

$$\tau = rF_T$$

The angular acceleration is related to the linear acceleration of the string by $\alpha = \frac{a}{r}$. You can measure a with the Smart Pulley.

The easiest way to create the necessary tension force in the string is to run it over the pulley and hang a known mass from it. The value of the tension force is related to the value of the mass by Newton's second law:

$$\Sigma F = F_T - Mg = -Ma$$

which can be solved for the tension force:

$$F_T = M(g - a)$$

You now have all the values you need to solve for the initial velocity of the ball:

$$v_o = \frac{\omega r^2 M(g-a)}{amR} \quad \text{or} \quad v_o = \frac{2\pi r^2 M(g-a)}{amRT}$$

You can then check this calculated value of v_o with a value you measure experimentally with photogates.

Procedure

In this lab, you will give a steel ball of mass m and velocity v by launching it from a spring gun. The gun is aimed at a catcher mounted at the end of a stationary rotary track. After it catches the ball, the track will begin to rotate about a vertical axis at its center.

Before it is caught, while it is flying with constant velocity v, the ball will have angular momentum about the track axis given by:

$$\mathbf{L}_{before} = mvr\sin\theta$$

Even though both **r** and θ vary as the ball approaches the track, the value of $\mathbf{L}_{before}$ is constant because $r \sin\theta = R \rightarrow$ the radius of the track.

You will measure the ball's linear velocity out of the launcher and the track's angular velocity after it catches the ball. You can then calculate the ball's initial speed by employing the angular momentum conservation law: $\mathbf{L}_{before} = \mathbf{L}_{after} = I\,\omega$, and compare your calculated speed to your measured speed. In the equation for $\mathbf{L}_{after}$, I is the moment of inertia of the system about the track's axis of rotation and ω is the angular speed of the system after the ball is caught.

With the power switched off, connect these sensors to the Interface:
1. Plug two Photogate sensors into Digital Channels 1 and 2.
2. Plug the Smart Pulley sensor into Digital Channel 3.

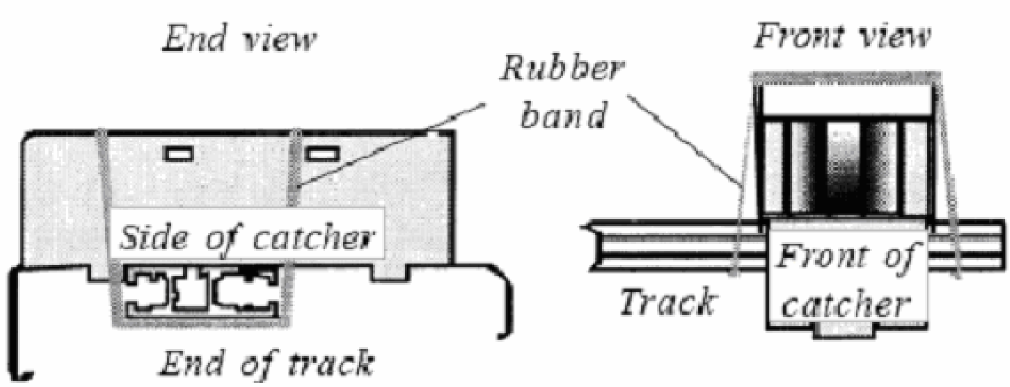


Figure 11.2: Mounting catcher on end of track

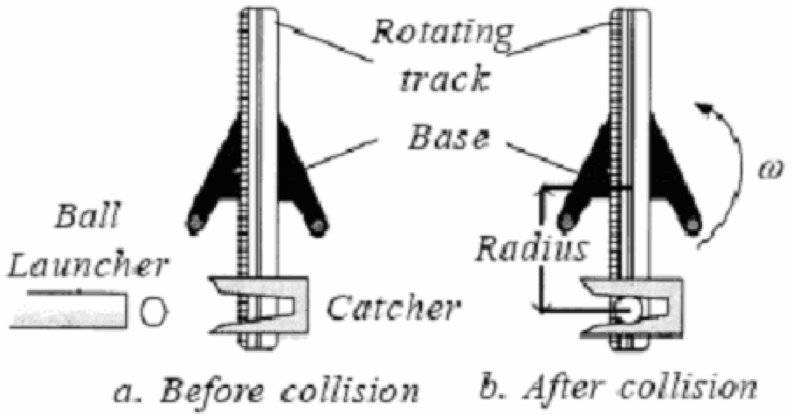


Figure 11.3: Top view of ball and track

A. Measuring *v* and *ω*

1. Clamp the base of the launcher to the edge of the table near the interface with the gun pointed along the edge. Adjust the launcher to be horizontal. Attach the photogate bracket to the launcher and mount photogate #1 at the first position on the bracket and photogate #2 at the second position.
2. Switch on the computer system, open Data Studio, and select Create Experiment. Open the Photogate sensor twice to show photogates connected to digital channels 1 and 2. Open the Smart Pulley sensor to connect to digital channel 3.
3. Open the Timer Setup window. Set the timing to start when the first photogate is blocked and set the timing to stop when the second photogate is blocked. Make the label read *Time between gates*.
4. Measure the distance d between the two photogate beams and enter this value (in m) in Table 11.1.
5. Open a Digits display and choose *Time between gates* as the data source. Open a Graph display and choose *angular velocity* (channel 3) as the data source.
6. Using a rubber band, attach the catcher to the end of the track, as shown in Figure 11.2. Slide the catcher and rubber band to the end of the track so that the catcher's sides are at the 18.0 cm and 24.0 cm points on the track scale (measure its position with a precision of 3 sig. fig., i.e., to the nearest mm).
7. Place the rotating track assembly at the table's edge so that the Smart Pulley extends beyond the edge and the catcher is in front of the spring gun when the track is perpendicular to the ball's flight path, as shown in Figure 11.3. Be sure that the rotating track does not strike the gun.
8. Level the base, and do not move the base during the remainder of the experiment.
9. Weigh the ball and record its mass m_b (with a precision of 3 sig fig) in Table 11.1.
10. Load the ball into the gun and cock it to the middle range position. Rotate the track so the catcher is in front of the gun and the track is stationary. Click Start, shoot the ball into the catcher, then click Stop.
11. Record the Time between gates t in Table 11.1, and use this time to calculate and record the launch velocity v. Record the angular velocity ω of the platform right after the ball is caught (maximum value of the graph).
12. Measure and record the ball's radius of revolution R (to the nearest mm) from the axis of rotation to the center of the ball in the catcher.

13. Repeat steps 10 and 11 two more times. Record the average of the three values of *v* and of the three values of ω (both with a precision of 3 sig. fig.) in Table 11.1.

B. Measuring *I* and calculating *v*

14. ***Leave the ball in the catcher***. Remove the rotary sensor and attach a smart pulley as shown in Figure 11.4. Unplug the two Photogate sensors and the rotary sensor, and plug the smart pulley into Digital Channel 4. Measure the diameter of the spool on the axis and record its ***radius*** *r* (with a precision of 3 sig fig) in Table 11.2.
15. Cut a 1-m length of string and tie a loop knot in one end. Poke the other end in the small hole in the spool on the axis and wind the string around the spool. Run the string over the Smart Pulley and hook a weight hanger in the loop. Be sure the string running from the spool to the pulley is horizontal.
16. Open a Graph display and select *velocity* (channel 4).
17. Hang 100 g from the string and wind the string on the spool until the weight hanger is just below the pulley. Hold the track to prevent it from rotating.
18. Press *Start* and release the track. When the weight hanger hits the floor, press *Stop*. The linear acceleration of the mass and string is the slope of the linear part of the velocity graph. Record the value of the hanging mass m_h and the value of the string's linear acceleration *a* in Table 11.2.
19. Use the linear acceleration *a* to calculate and record the angular acceleration α.
20. Repeat steps 18 and 19 two times. Record the average of the angular acceleration of the platform in Table 11.2.
21. Use your measured data to calculate the moment of inertia *I*. Record *I* (with a precision of 3 sig. fig.) in Table 11.2.
22. Using conservation of angular momentum, *calculate* the launch speed *v* of the ball. Compare your calculated and measured values of *v* by calculating and recording the percent difference between them.
23. Close Data Studio, disassemble the equipment, neatly coil and tie the sensor cables, return all equipment to the lab cart, and clean up your lab table area.

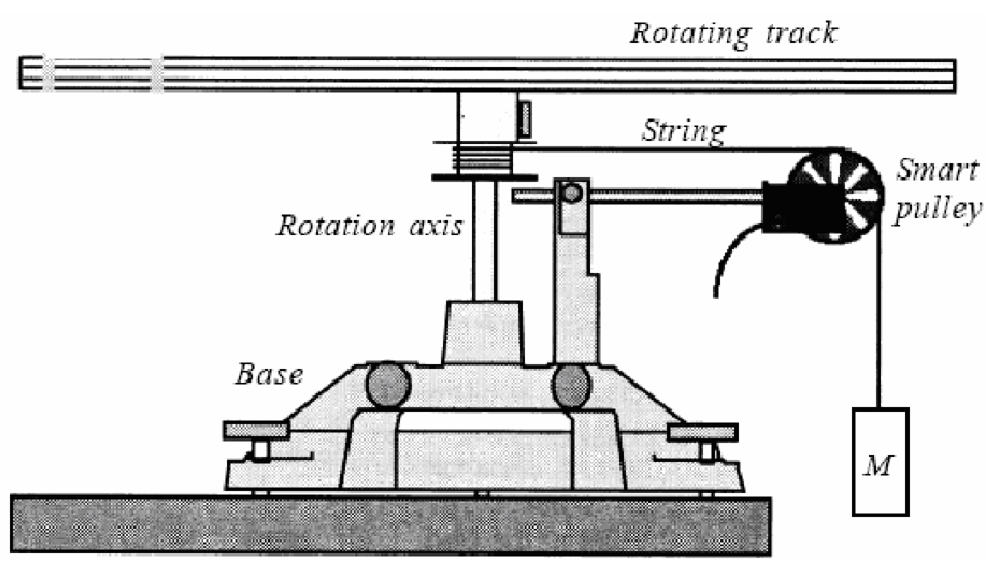


Figure 11.4: Applying a known force to create torque